AMSLER

ALFRED J. AMSLER & CO
SCHAFFHOUSE
(Switzerland)

Description No. 45/41

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AMSLER'S

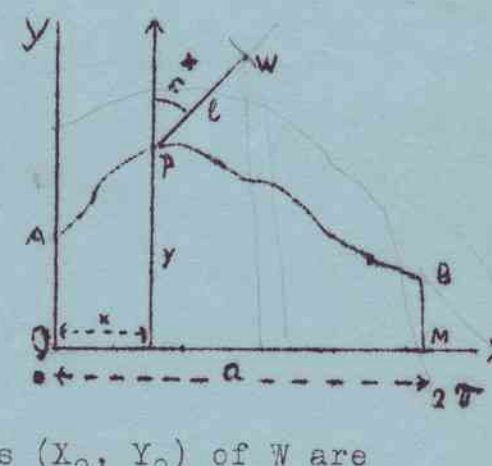
HARVEY HARMONIC ANALYSER

(See photographs 7290, 7291, 7292, 7293, 7294)

The instrument essentially is a simple type of Harmonic Analyser, embodying quite novel ideas and designed so that it can be used not only as a Harmonic Analyser, but also as Planimeter for evaluating areas and as Integrator for measuring the first and second moments.

PRINCIPLE. Let $y = f(x)$ be the equation to the curve to be analysed in a Fourier Series over the range $0-2\pi$, which may be represented by a length a (fig.1). Let the curve meet the ordinates $X = 0$ and $X = 2\pi$ in points A and B respectively. The area enclosed by the boundary O A B M O is represented by

$$\int_0^{2\pi} f(x)\, dx .$$



Let PW be a bar of length l having the end P over the boundary, and such that for an abscissa x of P, PW makes the angle nx with the positive direction of the y-axis. Then, when P is on the curve, the Coordinates (X_1, Y_1) of W are

$$\left.\begin{aligned} X_1 &= x + l \sin nx, \\ Y_1 &= y + l \cos nx, \end{aligned}\right)$$

and when P is on OM, the coordinates (X_2, Y_2) of W are

$$\left.\begin{aligned} X_2 &= x + l \sin nx \\ Y_2 &= \quad l \cos nx \end{aligned}\right)$$

If P is brought round the boundary, the area enclosed by the curve traced by W is

$$\int_{x=0}^{x=2\pi} Y_1\, d\, X_1 \text{ (along the curve) } - \int_{x=0}^{x=2\pi} Y_2 dX_2 \text{ (along OM)}$$

This can be shown to be equal to

$$\int_0^{2\pi} y\, dx \; + \; nl \int_0^{2\pi} y \cos nx dx$$

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or $$\int_0^{2\pi} f(x)\, dx + nl \int_0^{2\pi} f(x) \cos nx dx$$

Hence the area swept out by the bar PW alone is

$$nl \int_0^{2\pi} f(x) \cos nx dx$$

Now if a wheel be placed on PW at W, with its axis on PW, such that it can roll on the paper during the movement, and if r is the distance rolled by the wheel in the complete circuit of P round the boundary, the area swept out by PW is lr. If d is the diameter of the rolling wheel and m_1 is the number of revolutions as given by the counter, the area is $l \pi m_1 d$.

Hence : $$l\pi\, m_1 d = nl \int_0^{2\pi} f(x) \cos nx\, dx$$

$$\therefore \quad \frac{m_1 d}{n} = \frac{1}{\pi} \int_0^{2\pi} f(x) \cos nx\, dx$$

$$\frac{m_1 d}{n} = a_n$$ (Coefficient of cos nx in the Fourier Series).

It can be shown similarly that if PW makes the angle nx with the negative direction of x-axis, when P has coordinates (x, y) at a point on the boundary, the area bounded by the curve traced by W is

$$\int_0^{2\pi} f(x)\, dx + nl \int_0^{2\pi} f(x) \sin nx\, dx$$

and if m_2 is the number of revolutions of the wheel in the complete circuit made by P

$$\frac{m_2 d}{n} = \frac{1}{\pi} \int_0^{2\pi} f(x) \sin nx\, dx$$

$$\frac{m_2 d}{n} = b_n$$ (Coefficient of sin nx in the Fourier Series).

In the instrument d is equal to 2 cm. If all the readings of wheel are multiplied by 2 and divided by n, the results are the required coefficients measured in centimetres.

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THE INSTRUMENT.

The instrument is designed so that PW can make the angle nx with the +ve y-dirn. or the -ve x-dirn. for coordinates (x,y) of P, according as a_n or b_n is required, from n = 1 to n = 6.

A T-shaped frame C D E F (see photograph 7291) can move in the y-direction on 3 wheels R_1, R_2 and R_3, two of which, R_1 and R_2 are carried in the cross piece EF by axles to which and to the x-axis, the axis of R_3 is parallel. The wheels R_1 and R_2 run in the groove of a rail A A parallel to the y-axis. A bar B_1 B_2 is fixed to the frame and on its top surface is cut a groove G G and to one vertical face a rack KK ; both groove and rack being parallel to the x-direction. A trolley L is suspended by 2 V-shaped wheels V_1 and V_2 which sit on the edges of the groove, and a wheel V_3 which rolls on the bar C D. The trolley can be moved freely in the x-direction between stops S_1 and S_2, so that a tracing point T fixed to it can traverse a distance a in the x-direction representing the range $0 - 2\pi$. Thus T can be made to trace any boundary within the range ; there is no limit to the movement of T in the y-direction.

A bar H H in the y-direction can be drawn through guides under the trolley by a milled screw nut N. In H H there is a vertical hole in which a spindle P can turn. To the upper end of the spindle can be fixed one of a set of gear wheels of radii $\frac{a}{2\pi n}$, n = 1, 2 - - - 6.

To the lower part of the spindle P is fixed a small frame in which is set the integrating roller W, to roll on the paper, and with its axis passing through the axis of the spindle P.

To set the instrument, the trolley is brought up to the stop S_1 so that T is at the origin of coordinates, then by means of the screw nut N the gear wheel is drawn in to engage with the rack and so that the direction of PW is the +ve y-dirn or the -ve x-dirn, according as a_n or b_n is required.

As T is brought round the boundary, P will describe a similar boundary, and PW will make the required angle with the y or x directions, for a position (x, y) of T.

The curve to be analysed must be replotted on the period length a of the instrument. The base length a is exactly 24 cm.

Using the Instrument as a PLANIMETER. To employ the instrument as a planimeter (see photograph 7293), the trolley L is fixed by a catch to the bar B_1 B_2, so that the spindle P can

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be moved only in the direction allowed by the wheels R_1, R_2 and R_3. A tracing arm J T_1 carrying an auxiliary integrating roller W_1, the axle of which is parallel to the direction JT_1, can be fixed to the sliding bar H H. (The invardly toothed crown seen on the photograph plays no part in this operation). When T_1 has traced the boundary of a curve, the roller W_1 records the corresponding area. Thus the coefficient

$$a_0 = \frac{1}{2\pi} \int_0^{2\pi} f(x)\, dx,$$

of a Fourier Series can be found.

Using the instrument as a MOMENT INTEGRATOR.

Photograph 7293 shows the arrangement for finding the First Moment of an area about a line T U. The trolley is fixed to the bar B_1B_2 so that the line joining T to the pivot P is perpendicular to the axes of the wheels. The line T U passes through the axes of the spindle P and of two vertical holes I_1 and I_2 in the bar H H (see photograph 7292). On the spindle P is fixed a gear wheel (not to engage with the rack) the radius of which to that of the inwardly toothed crown is 1 : 2. A spindle J fixed to the crown at its centre is dropped into the hole I_1, so that the toothed crown and gear wheel engage, and at the same time the axis of the first integrating roller W should be perpendicular to J T_1 when T_1 is on the line T U. The roller W will record the First Moment of the area enclosed by a boundary traced out by T_1.

Similarly, for the Second Moment of the area about T U, a gear wheel is fixed to the spindle P, whose radius to that of the inwardly toothed crown is 1 : 3. The spindle J is dropped into the hole I_2 in H H, so that gear wheel and crown engage in such positions that the axis of the first integrating wheel w is in the direction J T_1, when T_1 is on the line T U. The reading of W in this case is associated with that for the area, simultaneously found from the auxiliary area-roller W_1, according to a formula, and the second moment of the area about T U is obtained (photograph 7294).

The theory underlying these operations is the same as for AMSLER'S ORIGINAL INTEGRATORS.

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The Instrument and the rail are supplied carefully packed each in a wellmade carrying case provided with lock and key.

Size of instrument case : about 53 x 37,5 x 10,5 cm
(about 21 x 15 x 4¼ in.)

Size of rail-case : about 75 x 8 x 6 cm
(about 30 x 3 x 2½ in.)

Weight of instrument in carrying case : about 5,4 kg = 12 lbs.

Weight of rail in carrying case : about 2,0 kg = 4½ lbs.

Weight of instrument and rail packed for despatch :
about 32 kg = 70 lbs.

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Supplementary attachment for Harmonics higher than the 6th

On special request and against an extra charge, the Analyser can be equipped with double multiplication gears with additional harmonic pinions, permitting higher harmonics up to the 14th order to be determined. Two such double multiplication gears with corresponding harmonic pinions set can be provided, the one for the even harmonics $n = 8$, 10, 12 and 14, the other for the odds $n = 7$, 9, 11 and 13. Both double multiplication gears and relative harmonic pinions may be ordered together.

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SCHAFFHOUSE
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Instruction No. 61/14.

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7/12/37.

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AMSLER'S HARVEY HARMONIC ANALYSER

Working Instruction

See: Description No. 45/41

Drawing No. 28883/317	- Instrument used as Analyser
Drawing No. 28312/317	- Instrument used as Integrator
Photograph No. 7290	- Analyser and rail, both packed in their wooden case,
" No. 7291	- Instrument used as Analyser for harmonics from n= 1 up to n = 6,
" No. 7292	- Instrument used as Analyser with double gear attachment for harmonics from n = 1 up to n = 10 and n = 12,
" No. 7293	- Instrument used as Integrator for static moment
" No. 7294	- Instrument used as Integrator for moment of inertia.

I. Using the instrument as Harmonic Analyser.

1.) Drawing the diagram to be analysed on proper scale.

The diagram to be analysed (generally on small scale) must be redrawn so that the abscissa length of a complete period (corresponding to 2π) is exactly equal to the base length of the instrument of 24 cm. Conveniently - but not necessarily - the two points of zero ordinate of the periodic wave line may be taken as ends of the analysed diagram portion.

The best way is to enlarge the diagram photographically, or by using a pantograph. The diagram may also be redrawn by hand dividing the original base length into 24 equal parts, measuring the 24 ordinates and replotting them to a convenient scale on 24 ordinates all 1 cm apart. This latter method however is less advisable, because when redrawing the enlarged diagram by hand, minute subtlenesses in the shape of the wave line, which are determinant for the higher harmonics, may easily be altered.

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For redrawing the diagram use strong Bristol or drawing paper, not subject to dilatation or shrinkage due to air moisture or temperature changes.

2.) Placing the Analyser.

Place the rail upon the chart of the enlarged diagram so that the rail is approximately perpendicular to the abscissa, the rail being left and the tracing point towards the operator as in drawing 28883/317, lower part. Depose the Analyser in such a way that the two wheels run in the groove of the rail, and the measuring roller rests on the chart. Affix the counterweight at the rear of the main carriage frame. Now shift the rail until the tracing point follows exactly the abscissa-line of the diagram and that when the sweeping-arm's carriage is in contact with each of the two stops limiting its 24 cm - stroke, the point exactly coincides with the ends of the wave period of 2π .

The set pin inserted through the wheel base of the main carriage and the rail is intended to prevent the main carriage from racing along the rail and falling off the table, when displacing the drawing board with the complete instrument laid out on it.

3.) Inserting the interchangeable pinions for the different harmonics.

For harmonics n = 1 to n = 6 use single pinions

For harmonics n = 7 to n = 10 and n = 12, use the double multiplication gear.

Every pinion is engraved with the number of the corresponding harmonic. The interchangeable pinions must always be inserted so that the numbers are on top.

The adaptation of the double multiplication gear for harmonics from n = 7 to n = 10 and for n = 12 is represented in the upper part of drawing 28883/317. The small pinions used with the double gear are meshing sometimes with the upper, sometimes with the lower large wheel on the countershaft. For inserting those pinions properly even in this case no other provision must be taken than placing them with the harmonic number on top, as the pinions are stamped on the correct upper face.

For inserting the interchangeable pinions proceed as follows:

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Bring the sweeping-arm's carriage in contact with the left stroke-end stop. Shift the sweeping-arm's axle to a distance from the rack somewhat larger than the radius of the pinion to be inserted by turning to the left the milled knob just behind the tracing point. Unscrew the milled screwnut on top of the sweeping-arm's axle and insert the pinion so that the arrow marked "C" is in line but opposite with the sweeping arm, as represented in drawing 28883/317, bottom, the set pins in the axle flange engaging with the 2 pin-holes in the pinion hub. Tighten again the milled screw nut.

For <u>cosine readings</u> the sweeping-arm's carriage being in contact with the left stop and the sweeping arm turned <u>vertically away from the operator</u> (position in full lines in drawing No. 28883/317), by turning the milled front knob to the right bring carrefully the pinion into mesh with the rack so that the arrow marked "C" of the pinion (C = cosine) is exactly opposite to the symmetric arrow on the rack.

For <u>sine readings</u> the sweeping-arm's carriage being in contact with the left stop, and the sweeping arm now turned <u>horizontally to the left</u> (position drawn in dotted lines on drawing No, 28883/317), by turning the milled front knob to the right bring carefully the pinion into mesh with the rack so that the arrow marked "S" of the pinion (S = sine) is exactly opposite to the symmetric arrow on the rack.

<u>IMPORTANT:</u> Proper adjustment of tooth pressure between pinion and rack must be made to personal feeling, neither too slack (danger of dead play!) nor too strong (danger of forcing the rack!), so as to ensure a smooth gearing of pinion and rack everywhere. After adjustment, check degree of contact by running the sweeping-arm's carriage to and fro several times the whole rack length.

With the double multiplication gear proceed as above replacing the word "rack" by "toothed wheel on the countershaft".

4.) <u>Circumscribing the wave line.</u>

Start from the initial point of the period <u>on the zero line</u> (point of ordinate = 0), the sweeping-arm's carriage being at left stop position, take the initial reading; circumscribe the wave line with the tracing point <u>in a left to right sense</u> up to the end of the period (sweeping-arm's carriage at right stop position), and trace back along the zero line from right to left to the starting point; take the final reading.

If the analysed portion of the wave line does not begin with a point of zero ordinate, but has a finite initial (and final) ordinate, then, starting from the zero line, follow first the initial ordinate up to the wave line, then the wave line

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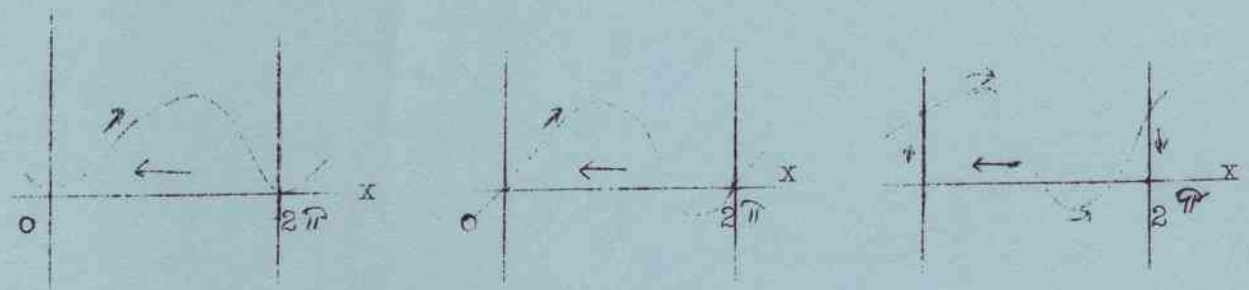
proper, then the final ordinate back to the zero line, and finally return on the zero line to the starting point.

The figures below illustrate some possible cases.



5.) Readings and calculations.

The instrument indications m_1 and m_2 for cosine and sine harmonics are the differences of the final and initial readings of the integrating roller after and before circumscription of the wave period in the above manner. m_1 and n m_2 are the numbers of revolutions of the integrating roller.

For accurate results, the circumscription of the diagram ought to be repeated several times in sequence , intermediate readings being taken at every return to the starting point, and the average difference formed.

The drum of the integrating roller is divided into 100 parts. Tenth of part are read on the vernier. Complete revolutions of the integrating roller are shown on a counting disc, which advances one line at each such revolution. The counting disc is arranged for total 10 revolutions of the integrating roller. A second counting disc which advances one line at each revolutions of the integrating roller.

Each complete reading is a figure of two digits and three decimals, the tens and the units being read on the second and first counting disc respectively, the first and second decimal on the drum of the measuring roller and the third decimal on the vernier.

When taking the second reading after the first measurement, be careful to ascertain whether the total motion of the measuring roller has been forward or backward and how many times and in what revolving direction the zero of the second counting

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disc has in fine passed the fixed index mark. Of the total travel of the measuring roller has caused more than one complete turn of the second disc, the figure 100 must be added to the difference of readings; this, however, will occur very seldom.

The difference between the final and initial readings, i e. m_1 and m_2 may be positive or negative according to the sign of the corresponding harmonic searched for. Therefore care must be taken of correctly determining and observing the sign of the instrument-indications.

IMPORTANT REMARK:

Due to inversion of the sense of rotation of the sweeping-arm by the double multiplication gear, the sign of the harmonics higher than the 6th becomes partly alterated, and the following rule is to be observed:

When using the double gear, sine readings give correct, cosine readings inversed sign, so that in the final result for the latter the sign found must be inverted.

Although the proper sign is given without possible misinterpretation by the sign of the difference of readings, it is recommendable for the observer to watch with the eyes the process of rotation of the roller so as to have a confirmation of the sign of the ultimate result. As for higher harmonics of certain curves the momentary number of revolutions of the integrating roller (not the final reading) becomes so considerable that adequate supervision of the advancing sense of the roller becomes sometimes very difficult, the second counting disc, if mostly not required for usual work, is of value for this purpose.

The wanted harmonics in cm or mm for the redrawn diagram of 24 cm period-length are deducted from the numbers of revolutions of the integrating disc for cosine and sine adjustment, m_1 and m_2, by the formulae:

a_n (coefficient of cos n x in the Fourier Series) $= \frac{m_1 d}{n}$

b_n (coefficient of sine n x in the Fourier Series) $= \frac{m_2 d}{n}$

where n is the number of the harmonic and d = 2 cm or 20 mm the diameter of the integrating roller (in the same unit as used for the harmonics).

For the original diagram before redrawing, the effective figures of the harmonics are obtained from the above ones by dividing by the magnification of the ordinate-scale. The enlargement of the abscissa scale to 24 cm period length does not in any

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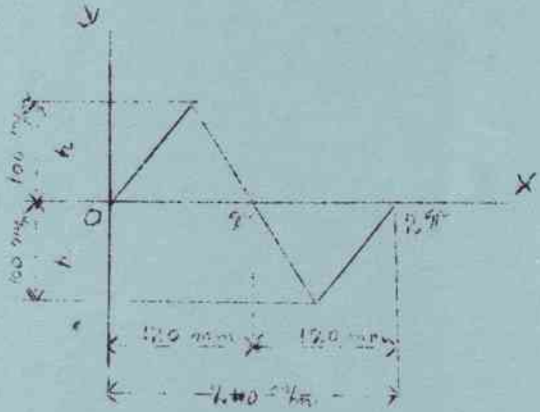
way affect the amplitude of the harmonics.

6.) <u>Example.</u>



Take as example the isosceles triangle-wave line, drawn on 24 cm = 240 mm base with a height h of 10 cm = 100 mm (amplitude). Choosing two points of zero ordinate as limits of the wave portion to be analysed, the readings and the calculation of the corresponding harmonics are found as below. The figure was circumscribed 5 times for each harmonic and the average of the indications has been taken. The unit adopted is the millimetre; thus d = 20.

1st Harmonic.

Cosine Readings	Diffe-rences	Sine Readings	Diffe-rences
constant		1,248	
"	0	5,302	+ 4,054)
"	0	9,345	+ 4,043)
"	0	13,393	+ 4,048)
"	0	17,436	+ 4,043)
"	0	21,483	+ 4,047)
average:	0	average:	+ 4,0470

n = 1

Fourier coefficients+

$a_1 = m_1 \times \frac{d}{n} = 0 \times \frac{20}{1} = 0$

$b_1 = m_2 \times \frac{d}{n} = + 4{,}0470 \times \frac{20}{1} =$

$= + 80{,}940$ mm

2nd Harmonic

Cosine Readings	Diffe-rences	Sine Readings	Diffe-rences
constant	0	constant	0
"		"	

n = 2

Fourier coefficients:

$a_2 = b_2 = 0$

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3^{rd} Harmonic.

Cosine Readings	Differences	Sine Readings	Differences
constant	0	104'400	- 1,357
"	0	103'043	- 1,352
"	0	101'691	- 1,351
"	0	100'340	- 1,350
"	0	98'990	- 1,343
"		97'647	
average :	0	average :	- 1,3506

$n = 3$

Fourier coefficients:

$a_3 = m_1 \times \frac{d}{n} = 0 \times \frac{20}{3} = 0$

$b_3 = m_2 \times \frac{d}{n} = - 1,3506 \times \frac{20}{3} = - 9,004^{mm}$

4^{th} Harmonic.

constant	0	constant	0
"		"	

$n = 4$

Fourier coefficients:

$a_4 = b_4 = 0$.

5^{th} Harmonic.

constant	0	15'308	+ 0,832
"	0	16'140	+ 0,821
"	0	16'961	+ 0,804
"	0	17'765	+ 0,799
"	0	18'564	+ 0,824
"		19'388	
average :	0	average :	+ 0,8160

$n = 5$

Fourier coefficients:

$a_5 = m_1 \times \frac{d}{n} = 0 \times \frac{20}{5} = 0$

$b_5 = m_2 \times \frac{d}{n} = + 0,8160 \times \frac{20}{5} = + 3,264^{mm}$

6^{th} Harmonic.

constant	0	constant	0
"		"	

$n = 6$

Fourier coefficients:

$a_6 = b_6 = 0.$

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7^{th} Harmonic.

(With double multiplication gear)

Cosine Readings	Differences	Sine Readings	Differences
constant		3,249	
"	0	2,676	- 0,573
"	0	2,104	- 0,572
"	0	1,538	- 0,566
"	0	0,966	- 0,572
"	0	0,390	- 0,576

$n = 7$

Fourier coefficients:

$a_7 = m_1 \times \frac{d}{n} = 0 \times \frac{20}{7} = 0$

$b_7 = m_2 \times \frac{d}{n} = 0,5718 \times \frac{20}{7} =$

$= - 1,634$ mm

(for cosine indications sign found to be inverted) (for sine indications sign found correct)

average : 0 average : - 0,5718

8^{th} Harmonic.

(With double multiplication gear)

constant	0	constant	0
"			

$n = 8$

Fourier coefficients:

(sign found to be inverted) (sign found correct) $a_8 = b_8 = 0.$

9^{th} Harmonic.

(With double multiplication gear)

constant		4,319	
"	0	4,768	+ 0,449
"	0	5,208	+ 0,440
"	0	5,649	+ 0,441
"	0	6,098	+ 0,449
"	0	6,547	+ 0,449

$n = 9$

Fourier coefficients:

$a_9 = m_1 \times \frac{d}{n} = 0 \times \frac{20}{9} = 0$

$b_9 = m_2 \times \frac{d}{n} = +0,4456 \times \frac{20}{9} =$

$= + 0,990^{mm}$

(sign found to be inverted) (sign found correct)

average : 0 average: + 0,4456

As a control of the above results, there existe incidentally, in the case of the isosceles triangle-wave line taken as example, a known development into a Fourier series:

$$Y = \frac{8}{\pi^2} h \left(\frac{\sin x}{1} - \frac{\sin 3x}{9} + \frac{\sin 5x}{25} - \frac{\sin 7x}{49} + \ldots\ldots \right)$$

with the general term: $\pm \frac{8}{\pi^2} h \cdot \frac{\sin n x}{n^2}$

for odds n = 1, 3, 5, 7 ...... etc.

Thus for the choosen amplitude h = 100 mm, the values of the Harmonics theoretically must be :

n =	a_n =	b_n =
1	0	+ 81,057^{mm}
3	0	- 9,006^{mm}
5	0	+ 3,242^{mm}
7	0	- 1,654^{mm}
9	0	+ 1,001^{mm}

which is in fair agreement with the mechanically determinded values.

II. Using the instrument as Planimeter and Integrator.

1. Placing the instrument.

Lock the sweeping-arm's carriage in the centre of the rack by means of the little target in middle of the carriage back.

Place the rail upon the drawing to be measured, this time parallel to the observer as in drawing No. 28312/317. Lay the wheels of the main carriage frame into the groove of the rail and affix the counterweight to the frame, just as before.

For using the instrument as a linear planimeter, no further provision must be made concerning the rail.

For use as an Integrator (determination of the static moment and of the moment of inertia) set the two gauges supplied with the instrument in such a way that their edges engage in the groove of the rail and their points rest on the drawing. Then move the rail backwards or forwards until the points of the

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gauges coincide exactly to the axis of moments x x (i.e. to that line with reference to which the static moment and the moment of inertia are wanted). The rail is then at the proper distance and parallel to the axis of moments.

2.) Adjustment for determining the area.

By turning the milled knob in front of the sweeping-arm's carriage, shift the broad head piece bearing the axle of the sweeping-arm to a large distance away from the rack. Insert the auxiliary tracing arm having the large rimmed invardly toothed crown as a centrepiece with its vertical pivot into the pivot-hole next to the sweeping-arm's axle, so that the auxiliary integrating roller rests on the paper.

This latter roller is the area-roller or "A"-roller.

3.) Adjustment for determining the static moment.

Insert the auxiliary tracing arm with its vertical pivot into the pivot-hole next to the sweeping-arm's axle, just as for the area. Bring the auxiliary tracing arm in alignment with the axis of moment, i.e. perpendicular to the rack. Turn the main sweeping arm vertically with the roller towards the observer (position in drawing No. 28312/317, bottom). Unscrew the milled nut on top of the sweeping-arm's axle and insert the harmonic pinion No. 2 upside down, i.e. with the stamped harmonic number at bottom, and push it home into mesh with the large invardly toothed crown so that the arrow marked "M_1" now visible on the pinion (M_1 = moment of 1st order) is exactly opposite the symmetric arrow on the toothed crown. Tighten again the milled screw nut.

With this adjustment the main sweeping arm is swept twice the angle of the auxiliary tracing arm, and the main integrating roller acts as static moment roller or as "M"-roller.

4.) Adjustment for determining the moment of inertia.

Insert the auxiliary tracing arm with its pivot now into the pivot-hole farthest away from the sweeping-arm's axle. Proceed just as for the static moment, only inserting now the third harmonic pinion upside down when the auxiliary tracing arm coincides with the axis of moments and the main sweeping arm is turned horizontally to the right (position in drawing No. 28312/317, top), and bringing it into mesh with the invardly toothed crown in such a position that the arrow marked "M_2" on pinion back ("M_2" = moment of 2^{nd} order) is exactly opposite the arrow on the toothed crown.

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With this adjustment the main sweeping arm is swept thrice the angle of the auxiliary tracing arm, and the main integrating roller acts in this case as inertia moment roller or as "I" - roller. Simultaneously with the "I"-roller, readings must be taken in the present case also from the auxiliary "A"-roller, fitted to the additional tracing arm as the formula for the moment of inertia is a two-termed one.

5.) Circumscribing the figure.

In addition to the fixed tracing point No. 1, the auxiliary tracing arm is provided with a movable (vertically sliding) point No. 2 giving a shorter tracing arm length, which is of value for measuring small figures. Wherever practicable circumscribe diagrams of small height with the moveable point No. 2 in order to obtain a greater travel of the rollers and consequently more accurate results than with the fixed point No.1.

Make a mark on the outline of the figure to be evaluated. Set the choosen tracing point on the mark and write down the readings of the graduated measuring rollers. Move the tracing point carefully along the outline of the figure from left to right in clockwise sense until it comes back to the starting position. Read again the measuring rollers and write down the readings under the corresponding first readings. Subtract the first readings from the second and write down the differences to the right of the corresponding readings.

The figures then express the travel of the respective rollers.

In the following formulae the

travel of roller	"A" (auxiliery)	is denoted by		a
" " "	"M" (main)	"	" "	m
" " "	"I" (main)	"	" "	i.

6.) Readings and calculations.

The drums of the integrating rollers are divided into 100 parts. Tenths of a part are read on the vernier. Complete revolutions of the integrating roller are shown on the counting disc which advances one line at every such revolution. The first counting disc is arranged for total 10 revolutions, the second counting disc for total 100 revolutions of the integrating roller.

Each complete reading is a figure of five or four digits, the tenthousands and the thousands being read on the second, respectively the first counting disc, the hundreds and

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tens on the drum and the units on the vernier. <u>The method of taking the reading thus is different to that followed when the instrument is used as Analyser.</u>

When taking the second reading after the first one, be careful to ascertain (just as in the case of the Analyser) whether the total motion of the measuring roller has been forward or backward, as the static moment may be positive or negative according as to whether the bulk of the figure is situated above or below the axis of moments, and similarly the second term denoted "i" to be subtracted in the two-termed formula for the moment of inertia may in certain cases have a negative value (and thus become in fact additive).

For accurate results the circumscription of the diagram ougth to be repeated several times in sequence, intermediate readings being taken after every complete cycle and the average of the differences then struck.

The area A, the static moment M and the inertia moment I are obtained from the differences of readings a and m of the area-roller and of the analysing roller adjusted for the first moment, respectively from the <u>simultaneous</u> differences of readings a and i of the area roller and the analysing roller adjusted for the second moment, by the following formulae:

For the fixed tracing point No. 1(long tracing arm)

Area:	$A\ cm^2$	$= \pi \times 0{,}04 \times a$
Static moment:	$M\ cm^3$	$= \pi \times 0{,}20 \times m$
Moment of inertia:	$I\ cm^4$	$= \pi \times \frac{4}{3} \times (3a - i)$

For the moveable tracing point No. 2(short tracing arm)

Area:	$A\ cm^2$	$= \pi \times 0{,}02 \times a$
Static moment:	$M\ cm^3$	$= \pi \times 0{,}0 \times m$
Moment of inertia:	$I\ cm^4$	$= \pi \times \frac{1}{6} \times (3a - i)$

7.) <u>Examples.</u>

Ø 15,205 cm

Given a circle of 15,205 cm in diameter. To measure: The static moment and the moment of inertia in reference to the tangent xx, the circle being on the <u>upper</u> side of the axis.

Thus with the harmonic pinion No. 2 and using the fixed No. 1 tracing point:

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Main integrating roller :

Readings	Differences	
6880		
	+ 2197	
9077		
	+ 2197	
1274		Average: m = + 2198,2
	+ 2198	
3472		
	+ 2199	
5671		
	+ 2200	
7871		

Static moment: $M = \pi \times 0,2 \times (+ 2198,2) = + 1381,21 \text{ cm}^3$.

Would the circle have been on the lower side of the tangent the subsequent readings would have been regressing instead of ingressing as above; thus the differences would have been negative. This shows that the static moment must now be taken as a negative quantity.

For the same circle and still the fixed No. 1 tracing point, using now the harmonic pinion No. 3:

Main roller

Readings	Differences	
9504		
	+ 1202	
10706		
	+ 1206	Average i =
11912		
	+ 1201	+ 1202,4
13113		
	+ 1203	
14316		
	+ 1200	
15516		

Auxiliary area-roller

Readings	Differences	
5199		
	+ 1445	
6644		
	+ 1444	
8088		Average
	+ 1446	
9534		a =
	+ 1445	
10979		+ 1445,0
	+ 1445	
12424		

Moment of inertia $= \pi \times \frac{4}{3} \times (3 \times 1445,0 - 120,24) = 13'115,1 \text{ cm}^4$

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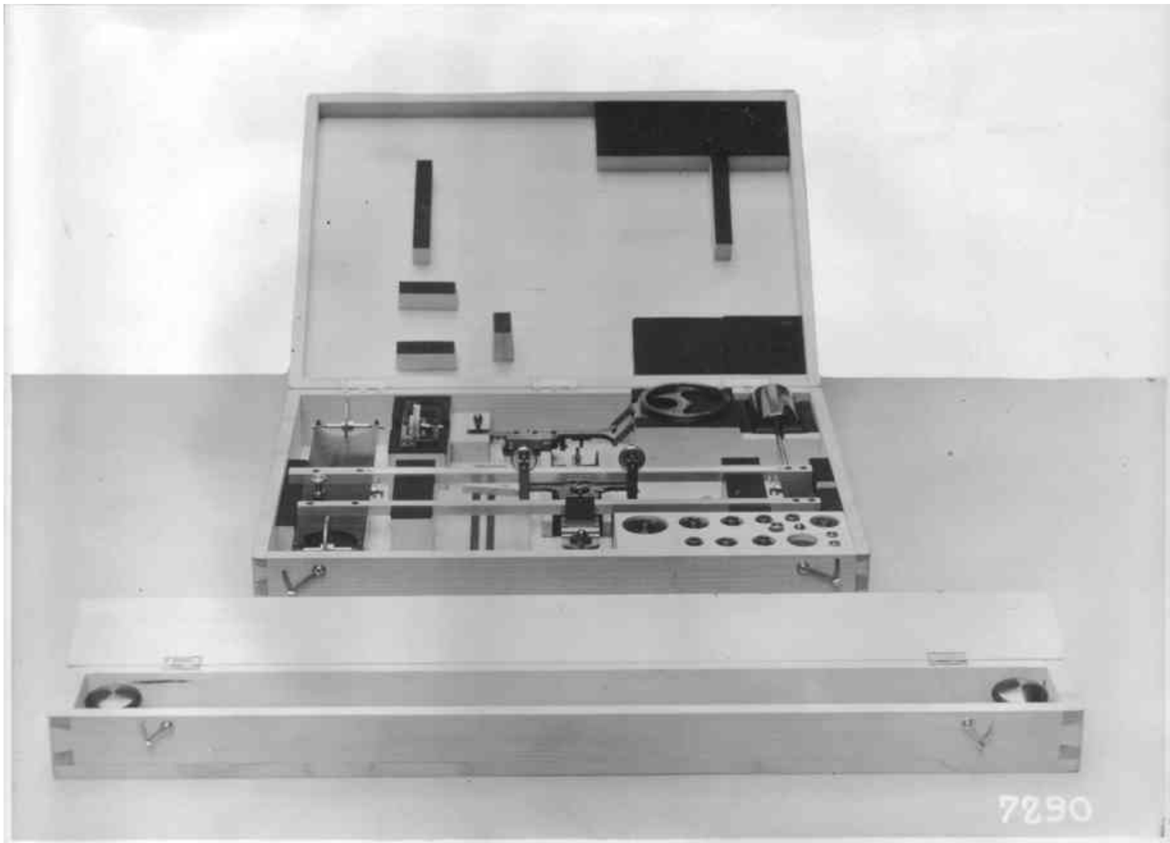
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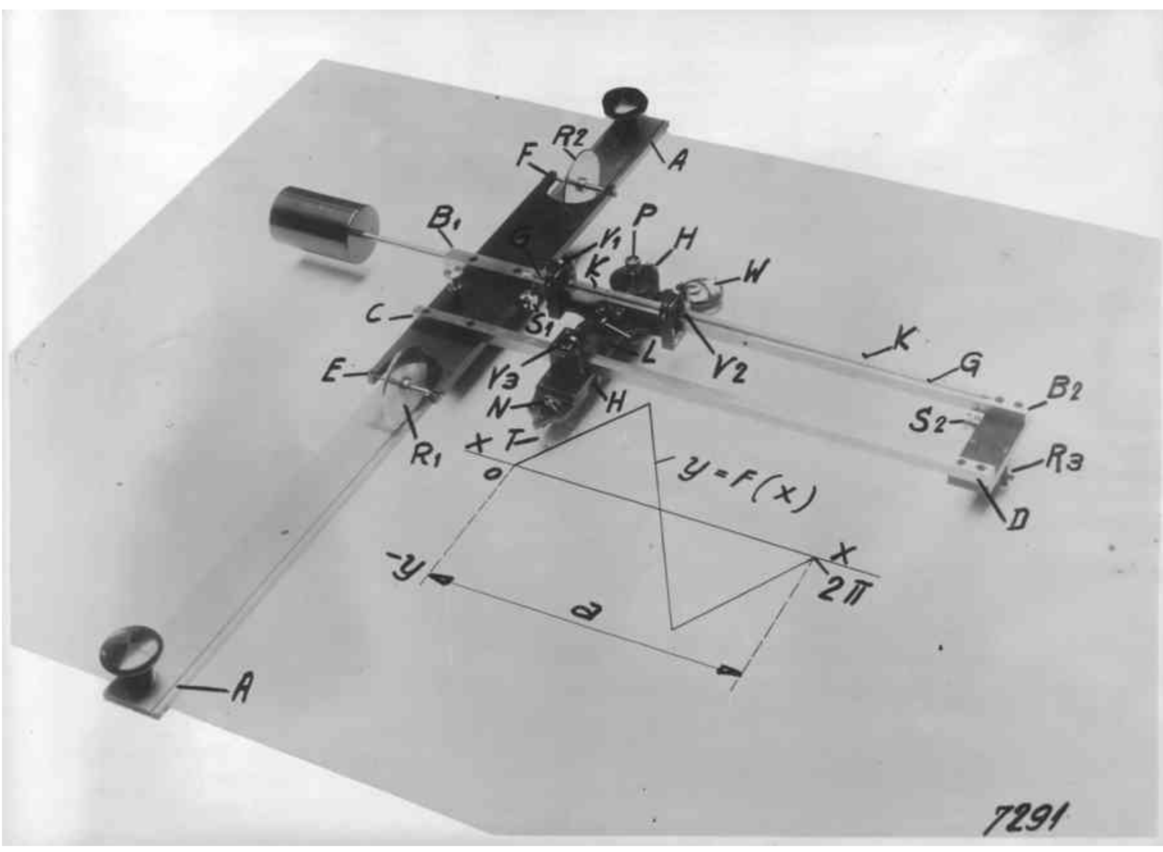
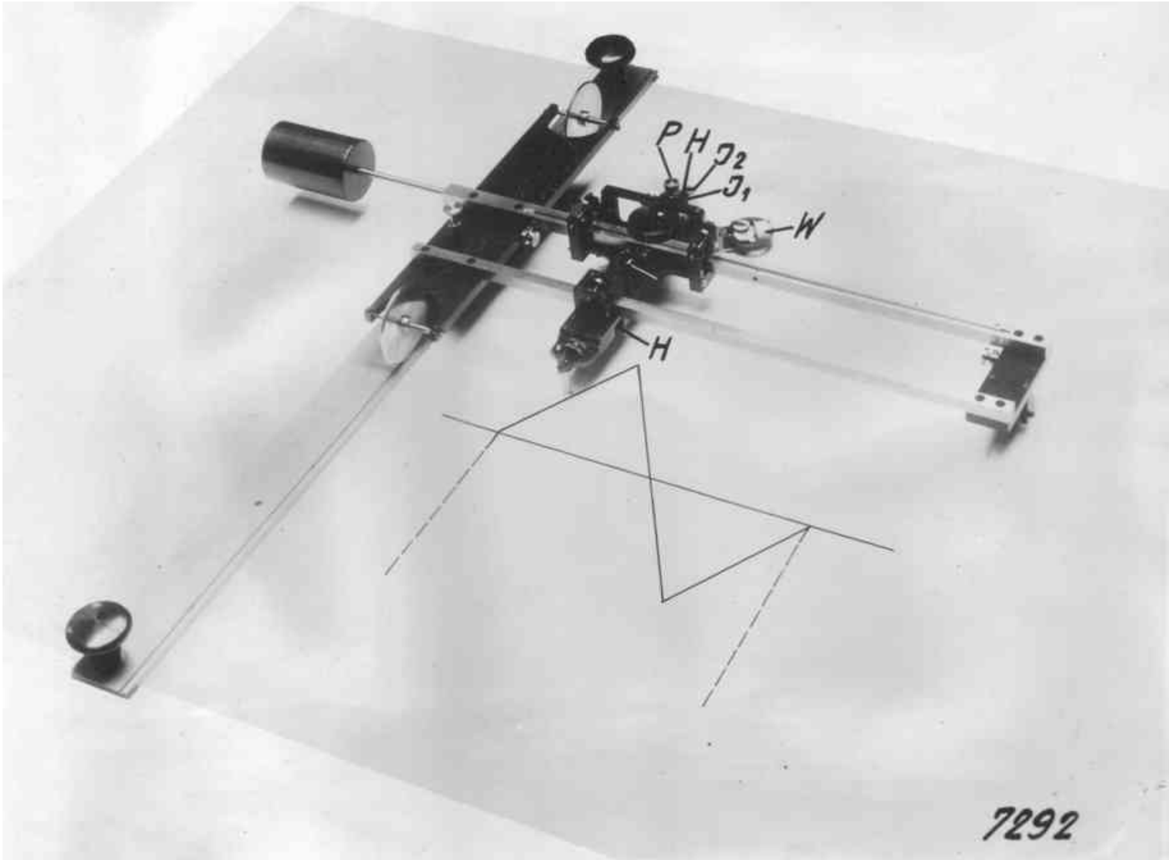
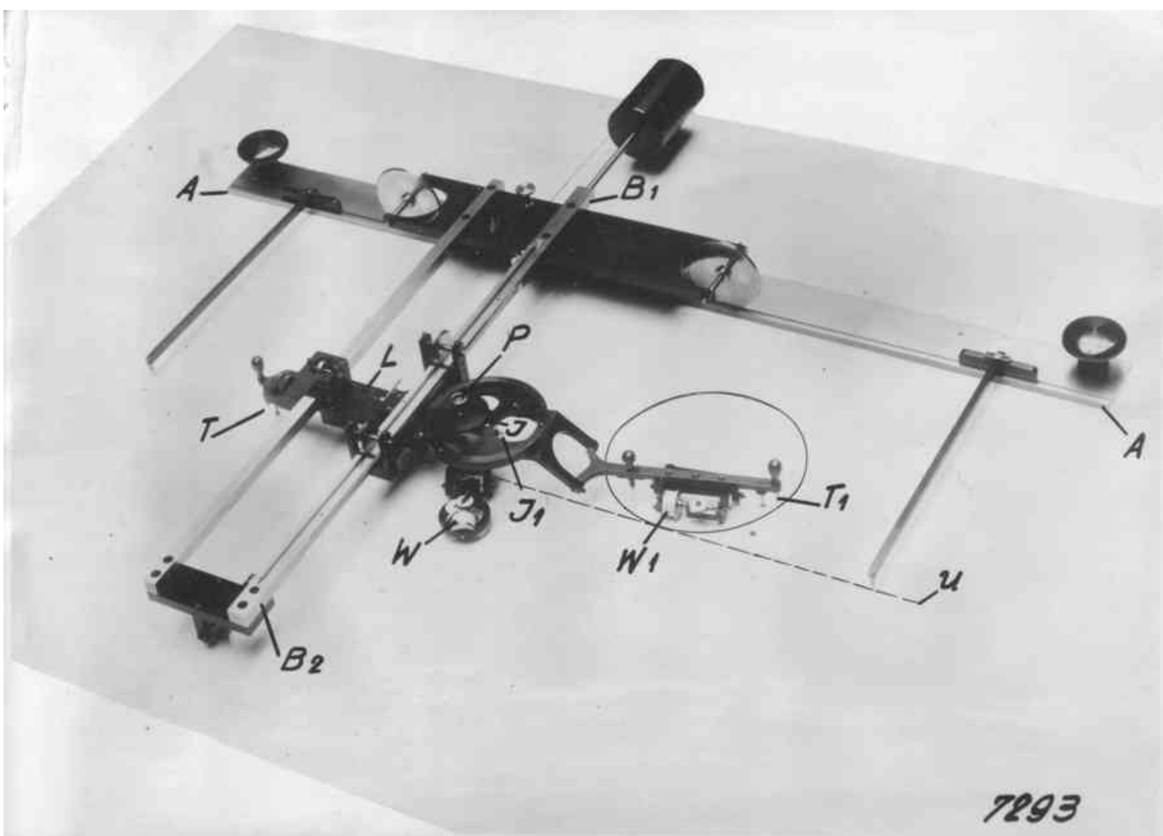
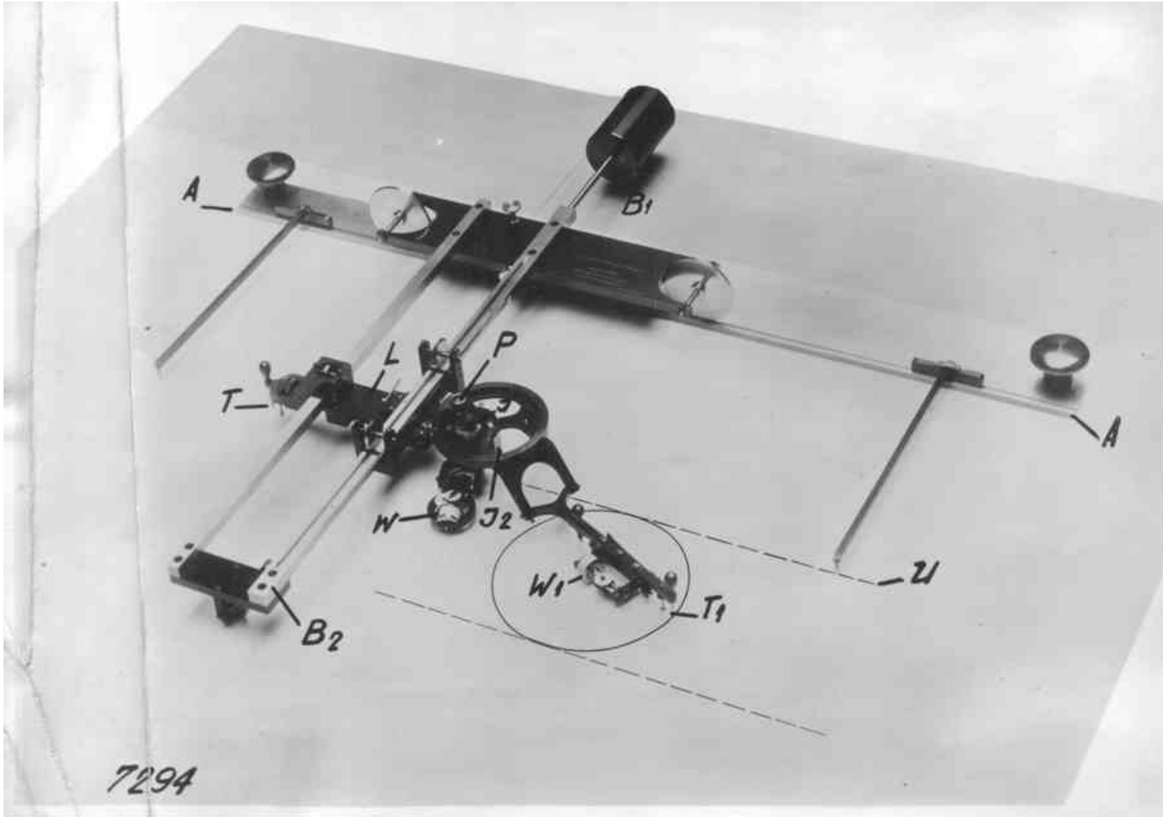

A
R2
F
B1
G
V1
K
P
H
W
C
S1
L
V2
K
G
E
V3
N
H
B2
S2
T
x
R1
o
y = F(x)
R3
D
-y
x
2π
a
A
7291


P
H
J2
J1
W
H
7292


A
B1
P
L
T
J
J1
W
W1
T1
A
u
B2
7293


A
B1
A
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W1
T1
B2
U
7294