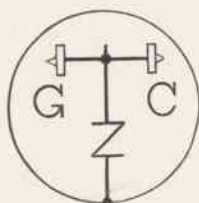


INSTRUCTIONS
OF USE FOR THE
ROLLING SPHERE
PLANIMETER



G. CORADI, ZURICH



DIRECTIONS

for the use of the

ROLLING SPHERE PLANIMETER

Preface. It is supposed that the owner of the Planimeter has full connection of the general and special theory of planimetry instruments, in this case of the rolling sphere planimeter.

Description. We therefore can begin with the special description of the rolling sphere planimeter itself and the manner to use and test it.

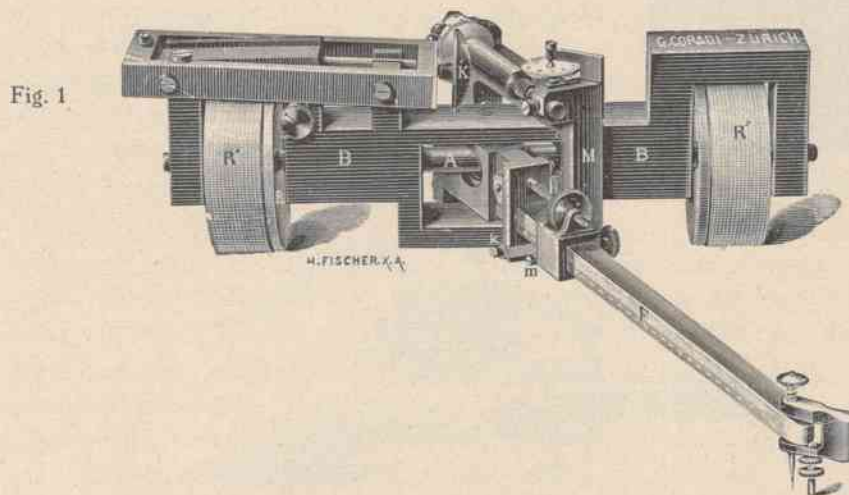


Fig. 1 represents the instrument in about half its size.

The guide line of the pivot of the tracer arm F is a straight line. The instrument rests on the plan at three points, the two rollers R' and the tracer F or its support s. In the frame B the axle A with fine hardened steel pivots works in two centre screws which have their threads in the frame B. The right hand screw can be adjusted and like all adjusting screws in my planimeters can be fixed by a brass set screw. The two cylindrical rollers R' are rigidly connected with the axle A, they are of equal diameter, concentric to the axle A and provided on their circumference with a kind of dotted milling in order to increase the accuracy of their parallel motion on the plan.

On the face of one of these rollers is a wheel with fine teeth. In it gears a small toothed wheel (not shown in the drawing) which is fixed on the steel axle of the spherical segment K. The axle is supported in a horizontal frame, on the left by a highly finished hardened pivot in an adjustable centre screw and on the other side by a hardened cone in a hardened steel plate. Outside this plate on a cone of the axle a spherical segment K is fixed, its axes being coincident with that of the axle. It is made of a hardened metal alloy not liable to oxidation. The left part of the frame of the axle and consequently the sphere itself can be somewhat raised on turning about horizontal pivots engaging in the frame B. It falls by its own weight until the small wheel of the axle rests on the wheel of the cylindrical roller whereby the proper gearing is automatically secured. By a half turn of the screw, marked with an arrow, the small wheel can be disengaged, if the arrow on the screw points downwards, the small wheel again gears.



The axle A and the axle of the sphere are parallel and in the same vertical plane, at the centre of the frame B, the vertical axle of rotation of the tracer arm is fixed. This axle consists of two adjustable steel screws fixed in the frame, the hardened pivots of which engage in two centres which are bored into the sleeve of the tracer arm in such a way that their line of connection (i. e. the axis of the tracer arm) is rectangular to the tracer arm, and that the imaginary vertical plane passing through the axis of the tracer arm and the point of the tracer, is parallel to the tracer arm.

The tracer arm adjustable in a sleeve, carrying the vertical axis of rotation, is by this enabled to vary its theoretical length l . The graduation on $\frac{1}{2}$ millimeters is not for measuring areas, but only for the purpose of determining accurately to $\frac{1}{20}$ mm, by means of the vernier at the sleeve, the relative length l of the bar in order that a revolution of the roller may correspond to a prescribed unit of area. The sleeve is made with a silver plated bevelled edge to facilitate the adjustment of the tracer arm to special marks of certain scales and values of the vernier unit. The exact adjustment is done by means of the vernier and the graduation.

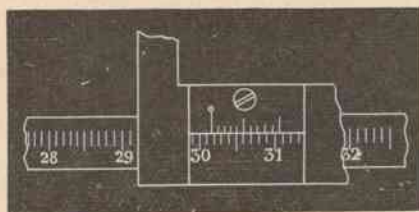


Fig. 2

Fig. 2 gives as setting of the vernier of the tracer arm the figure 301,5. For the accurate setting to the figures given in a table in the lid of the case the micrometer screw is used which is fixed in a shorter sleeve embracing the tracer arm: first the clamping screw of this sleeve is tightened and then the micrometer screw turned until the vernier reads the required figure and then the clamping screws of the sleeve of the tracer arm are tightened.

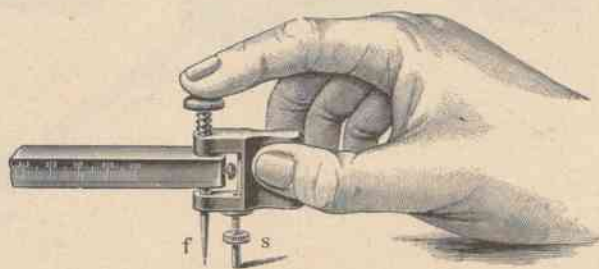


Fig. 3

Illustration 3 is shewing the tracer, its handle and support. The tracer f , pointed at its lower end, is vertically fixed by a set screw in the right end of the tracer arm. A handle turns easily about the vertical shaft and carries a support s , consisting of a pin with rounded foot.

Between the handle and the knob of the tracer a spiral spring keeps the point of the tracer just clear of the paper. The handle is grasped with the thumb and middle finger. The index finger thus free presses slightly the knob of the tracer into the paper to mark the starting point accurately or to keep the tracer fixed whilst the hand is sliding to a new position on the paper.

The frame M carrying the cylindrical measuring roller catches under the tracer arm at the front and back and can turn about a horizontal axis parallel to the tracer arm. The axle consists of two adjustable screws in the frame M, the points of which work in the centres in the sleeve of the tracer arm. The centre point turned towards the tracer is inserted in a small steel plate laterally adjustable by the two screws K, in order to bring the measuring cylinder into a perfect parallel position to the tracer arm.



A spiral screw suspended at the frame M on the one side and at the tracer arm sleeve on the other, draws the frame M up against the spherical segment.

A screw with cylindrically milled head in the frame M which presses against the tracer arm enables the frame M to be moved gently away from, thus bringing sphere and cylinder out of contact.

This little movement of screwing should always be made when the instrument is set out of work and equally when it is replaced in its case, in order to prevent any jar of the cylinder against the sphere and injury of these two most delicate parts of the instrument. For this purpose it is recommended to place a cushion of silk paper or a small square of soft leather between sphere and cylinder.

On the right side of the frame B a small fixing screw on the roller A is, when giving a slight pressure, preventing each non intended movement of the whole instrument, accidental starting and injury.

The tracer arm effects an angular motion of about 30° left and right of the base, while the magnitude of the direction of the base is unlimited. Consequently to this principle the instrument can in one only operation measure areas of unlimited length and of a width equal to the length determined on the tracer arm.

The Measuring Roller, the Counting Wheel and their Divisions

The principle on which the Rolling Sphere Planimeter is working is the straight line, mathematically true generated by the diameter of the cylinder framed parallel to the tracer arm and bearing on one extremity the measuring roller and when touching the sphere determines the radius of a circle of contact between the sphere and the cylinder. This radius increases proportionally to $1 \sin a$, so that the turning of the cylinder equals $J = C \cdot \pi \cdot \sin a$.

As the total of revolutions of the cylinder on the sphere produces the result the surface of these important parts of the planimeter must be kept absolutely metalically clean. The axle of the cylinder consists of a white metal alloy of sufficient hardness which does not easily oxidize.

The measuring roller situated on the forward directed end of the cylinder is a white celluloid drum accusing 100 revolutions. Very close to this drum, but not touching it, is placed a fixed cylindrical segment of the same material, carrying the vernier which subdivides each of the 100 intervals of the main graduation into 10 parts, so that a thousandth of a revolution of the roller can be read. The axis of the roller carries a worm, in which a counting wheel engages, furnished with 50 teeth and whose celluloid dial shows equally 50 intervals; at each revolution of the measuring roller the pointer on the dial advances one interval. Each interval of the dial therefore represents 1 revolution of the roller or 100 units of the vernier.

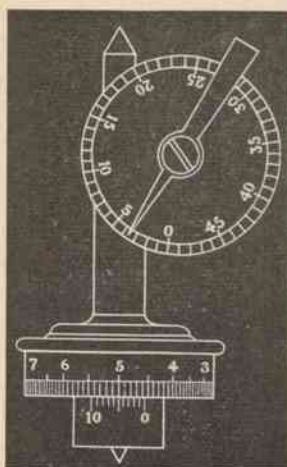


Fig. 4

Fig. 4 illustrates the roller and dial giving as an example the following reading: the index on the dial stands between the 3rd. and 4th. division, consequently 3 full revolutions of the roller, starting from 0, have been performed.

The first figure of the reading is therefore 3000

The zero of the vernier stands between 4 and 5, consequently from the zero of the roller 4 full hundreds only have been turned, the second figure is therefore 400

Of the ten lines lying between 4 and 5, the 5th. line has passed the zero point of the vernier.

This gives as tens 50

The 5th. line of the vernier exactly coincides with a division of the graduation, this gives the unit 5

The whole reading is therefore 3455



The cylindrical steel pins carrying the axle of the measuring roller enter into a brass frame and are kept in place by a steel set screws.

A steel screw, parallel and close to the bearing pin engages with its flanged head in a notch of the steel pin and enables the pin bearing to be adjusted in the direction of the axle of the roller.

No screwing is to be done at the roller bearing. With careful handling the instrument will not vary for tens of years. The roller is so adjusted, that it turns freely at the usual temperature of the room, when resting on the working surface.

Table in the lid of the case

With each planimeter a table of the following form is supplied and fixed in the case. This table is containing the length of the tracer individually determined for each instrument as well as for the values of the vernier for 4 or 5 different scales.

Scales	Position of the vernier on the tracer bar	Value of the unit of the vernier on the measuring roller
		(1 : 1)
1" = 50'		8 □' 0.0032 □"
1" = 25'	" " "	2 " 0.0032 "
1" = 100'		30 " 0.003 "
1" = 40'		4 " 0.0025 "
1" = 100'		20 " 0.002 "
1" = 50'		4 " 0.0016 "
1" = 40'		2 " 0.00125 "
1" = 30'	" " "	1 " 0.00111... "
1" = 100'		10 " 0.001 "
1" = 200'		20 " 0.0005 "

Always trace the
outline slowly

The sphere and cylinder should always be kept absolutely free from dust etc.

Care should be taken that the cylinder is moved gently when bringing it into contact with the sphere.

When the planimeter is not in use a pad should be placed between the cylinder and sphere.

Figures of considerably greater length than breadth should be placed at an angle to the X-axis.

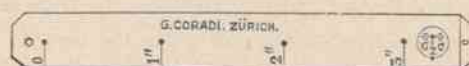
Zurich, the 19 No. G. Coradi.

The first vertical column marks the scale to which the plan of the area is drawn. In the second column the length of the tracer arm is given in divisions of the vernier on the tracer arm. The third column contains the value of the unit of the roller, or the factor by which the turning u of the roller obtained in such units of the vernier by going once round the contour of the area, must be multiplied in order to have as result the area expressed in square feet. In the same column the values of the unit of the vernier are given in square inches on the scale 1:1.

The Testing Rule

Each planimeter is supplied with a small rule divided in 8 or 10 cm; or for English measure in 3" or 4".

Fig. 5





At the zero of the graduation a small hole is bored through which a needle point passes, kept in place by an overlapping screw.

Each line of the graduation is marked by a small conical hole, in which the point of the tracer reposes. The needle point is pressed cautiously in the paper until the rule lies flat upon. We will now in first line screw up the support of the tracer and place the point into one of the holes and by turning the rule round the point of center the tracer arm will describe the circle of a known radius.

The bewelled end of the rule is forming an index; the index line will confront the starting point of the intended circumscription which will be marked on the paper.

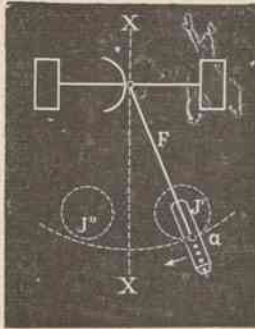


Fig. 6

Fig. 6 shows by a diagram how to effectuate the testing of the uniformity of the turning of the measuring roller by means of the testing rule.

In order to keep the measuring roller perfectly free from all translated pressure of the hand during the contourment (radial pressure) the tracer arm has to be loaded close the tracer with a suitable weight of lead or the like and the testing rule is guided instead of the tracer by its knob.

The testing rule is in certain cases advantageously replaced by a testing disc, made of a nickelled brass plate about $1\frac{1}{2}$ mm thick. It is pressed against the plan and kept thereon immovable by means of two steel pins projecting from its lower face.

In its surface perfectly flat circle-lines of 1", 2" and $2\frac{1}{2}$ " radius are engraved, in which the tracer point is carried. It is not necessary to mark the starting point of the tracer if it is chosen in that position where the tracer arm or its extension passes through the center of the circle since at this place the roller changes its sense of turning.

Directions for the use of The Rolling Sphere Planimeter

Before employing the planimeter be sure that it is in all parts in good condition, especially the axle of the measuring roller be examined to see that it revolves freely if gently touched.

No other geodetic instrument is so delicate as the planimeter and any injury, though not evident from external appearance of the instrument but only recognisable by a fine sense of mechanical movement, will have a detrimental effect on the results.

With regard to the good conditions the following rules should be born in mind:

By placing a book or a block underneath the frame B the latter is placed in such a position that the roller A can turn freely. The small wheel on the sphere axle is disengaged by lifting the releasing screw so that the arrow points upwards. The frame M of the measuring roller is by means of its releasing screw turned away from the sphere and the tracer arm is placed rectangular to the pole-arm.

Then we try slightly to displace the roller A in the direction of the axis, a slight, not noticeable play there of is of no consequence but the roller must turn easily and, if once made to rotate, not directly cause the movement.

The tracer arm is fixed in its sleeve by the set screw. It must turn easily about its vertical axis but without any noticeable play of the latter. This is ascertained by trying to move the tracer arm fixed in its sleeve to and fro about its longitudinal axis. The axis on which the spherical segment is fixed must turn easily and may have a very slight play in the direction of its axis; the horizontal axis of rotation of the frame of the measuring roller and that of the frame of the axle of the sphere must have no play. Such play is ascertained by trying to move these parts to and fro in the direction of their axis, the frame B being fixed. If the movements are too stiff or the play excessive all this can be altered by lightening or slackening the center screws, in doing so first slacken the brass set screws clamping them and tighten again after adjustment.



Some Rules for Measuring Operations

The sheet of paper containing the figures to be measured is placed on a level, well planed approximately horizontal board. If the paper is of such a nature that it lies perfectly flat on the board it need not to be fixed and can be placed together with the instrument resting thereon in a suitable position for the tracing in a good light.

Rolled plans must of course be smoothened out or flattened by placing weights, rulers, books and the like thereon.

As with the rolling planimeters the rollers cannot always be prevented from passing over the edge of the paper we therefore fix a sheet of the same paper close to the plan so that the edge can be passed over by the roller without jar.

In order to have done by the instrument at once the most positive work obtainable it is to retain in mind that before proceeding with the effective circumscribing of the area to be measured the instrument is to be brought in a suitable position with regard to the boundaries of the figure which should not pass close nor parallel to the base.

Fig. 7

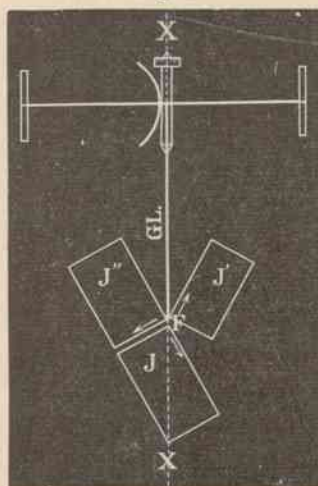


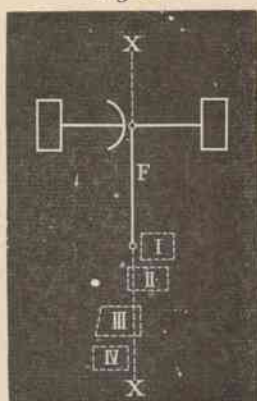
Illustration 7 shows some classical reciproc positions of the area and the instrument which can be ascertained without lengthy experiment by the following rule:

Place the tracer and the roller so that they are at a right angle to each other. With this mutual position (i. e. on the base) move the tracer on the boundary of the figure at a and start the motion of the tracer from this point, because on the base the roller makes the last movements and consequently errors in returning the tracer to the initial point will be more easily avoided.

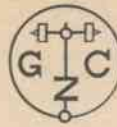
Here the tracer is taken along the base XX with the tracer arm in its normal position. The axis of the rollers will remain constantly parallel to the direction of motion of its point of contact and the roller will effectuate no indicating revolutions but only slip. The line which the point of the contact of the roller describes whilst the tracer travels along the base is therefore called the slipping line. If the guide line is a straight line the base and the guide line coincide or are parallel straight lines. If the roller turns on a sphere the slipping line is on the pole of the sphere and consequently forms only a point or a small circle, the radius of which equals the vertical distance between the axis of the sphere and the axis of the rollers.

If the plan to be measured is drawn, say to the scale 1:500, we select a length of the tracer arm which gives in the table in the case of this scale a convenient value of the vernier unit. As a rule the tracer arm is taken short for small scales 1:5000, 1:4000 etc. and long for large scales. If the table gives for the above scale the figure 266,6 then the tracer arm length is set into coincidence by means of its graduation and vernier. The value of the vernier unit will in this case be two square meters.

Fig. 8



Before going on to a rapid tracing of the area we convince ourselves that it can be effectuated without any difficulty. The axle of the sphere segment is thrown out of gear by turning the releasing screw under the frame of the roller so that the whole registering apparatus remains moveless during the first trial tracing, whereby especially with large areas a saving in wear and tear is effected. On lowering the frame of the sphere it may happen that the toothed wheels do not perfectly engage, we then work the instrument somewhat to and fro before commencing the tracing.



The tracer is placed at the initial point of the contour (on base XX) now let us know (instead of turning the roller to zero what would be logical in practise) as first reading L^1 and the indications at the measuring roller and the counting wheel give them just. Let this f. i. be the figure 3455 as in the illustration page 3. We then move the tracer point exactly clock wise along the contour of the area until it returns to the starting point, when, whilst keeping the tracer at this point we take the second reading L^2 on the measuring roller and counting wheel, let this be 9711. Now if the value of the vernier equals f the superficial area of the figure J equals $(L^2 - L^1) f$. In our supposition where f equals 2 square meters we in consequence obtain:

$$L^2 = 9711$$

$$L^1 = 3455$$

Difference $L^2 - L^1 = 6256$, multiplied by two square meters, gives as the area of the contoured figure 6256×2 square meters = 12 512 square meters. The operation is the same when made for English measure as follows:

Let in this case the plan of which the areas are to be measured be drawn on the scale of 1" to 100" and the individual areas not possess any dimensions exceeding 8", then we take the setting 215,0 given in the table as value of the vernier unit 100 square feet.

The reading before and after the tracing be, as before

$$L^2 = 9711$$

$$L^1 = 3455$$

Difference $L^2 - L^1 = 6256$ multiplied by 100 square feet gives 625 600 square feet as the area.

Owing to the play which the counting wheel has in the worm of the axle of the roller, in order that the movements of the latter are not impeded the index of the dial does not always point exactly to a division when the zero of the graduation coincides with the zero of the vernier.

If with the finger we move the dial slightly to and fro as much as its play permits, we shall soon see from the middle position of the pointer what division of the dial must be taken as the first figure of the reading.

Any error of 100 vernier units can however be easily avoided if the following rule is observed:

If the vernier on the measuring roller indicates below zero, say 80, 90 the preceding division of the dial is to be taken in consideration, if the contrary the vernier points beyond zero, say 10, 20 the division of the dial to which the index points is the first figure of the reading.

During the whole process care must be taken that while the tracer moves in the direction of the contourment the carriage does not deviate from the straight path by jolts, obstacles or inclination of the board.

The operation may be repeated in order to check errors in the reading, to decrease any errors due to inaccurate return to the initial point or to increase accuracy.

The guiding of the tracer is done by looking at it in the direction of the intended motion because in this case any lateral deviation will be most easily observed. If the point of the tracer arrives at a sharp turn of the contour or at the end of the line it is pressed down by the index finger so that without displacing the tracer the guiding hand can be placed in a convenient position.

For the tracing of straight lines a ruler is frequently used by certain operators. No greater accuracy however, in our opinion, or time saving is obtained, a careful tracing by free hand produces equal deviations to the right and to the left which in the final result compensate each other, while by using a ruler a constant error is frequently made even if the point of the tracer is accurately adjusted on the line. Owing to the lateral pressure of the ruler on the tracer and to the elasticity of the latter, the tracer and tracer point will not maintain their true position corresponding to that of the tracer arm.



Testing of the Rolling Sphere Planimeter

If the delicate parts of the instrument have been injured even the most accurate observation of the indications given for the testing of the instrument are futile and it is quite impossible to obtain good results from, or form a correct opinion of the instrument. We presume therefore that the organic positions of the planimeter to be tested are in perfect order.

Since a thorough testing requires the tracing of a considerable number of contours and as no error should be made during the tracing the mechanical mean of the testing rule is advantageously used for this purpose; it has been already described and is seen in its application from fig. 6.

The testing disc is equally frequently used and recommended.

Errors in tracing are however, on using mechanical means, not absolutely impossible. If during the movement of the tracer in a circle whose radius is determined by the testing rule the pressure of the knob of the tracer is not always tangential, then also the tracer point cannot have the circular line, the elasticity of the tracer and rule causes the tracer arm to assume a different position from that corresponding to the tracer.

This especially with large circles results in very considerable errors, as anyone can ascertain by observation and calculation. It is however desirable to place a small weight on the tracer and another on the center of the testing rule and to guide the testing rule itself instead of the knob of the tracer.

The testing rule is therefore especially intended for examining the uniformity of the results obtained from different tracings and with different positions of the pole; whilst the definite length of the tracer arm is determined by tracing figures of accurately known areas (squares and their dissections into triangles).

The testing must embrace seriatim the following points:

- 1) Whether in general the instrument is in a proper condition.
- 2) Whether the graduation on the roller is correct and central. Observe the vernier at different parts of the graduation at every ten divisions all round to whether the zero and ten of the vernier exactly correspond in all points to nine divisions of the graduation of the roller.
- 3) Whether the difference of the reading ($L^2 - L^1$) remains the same with repeated tracing of the figure. Fairly large differences up to 10 units of the vernier may occur between the various tracings, but these differences are more of an accidental nature, since they do not always occur at the same place of the roller, as in the case with polar planimeters, and are principally due to mistakes which, as stated before, are not even impossible with the testing rule.

If, for instance, in tracing a circle of 8 cm radius, the tracer arm is by lateral radial pressure, owing to the elasticity of the tracer caused to deviate even as little as 0,02 mm from its normal position, a deviation of $\frac{1}{2000}$ th. of the area = 10 units of the vernier is therefore occasioned. If we consider the very little force required to produce a lateral deviation of the tracer of 0,1 mm no one can help seeing the truth of our statement.

The same applies to the test whether the planimeter on tracing a circle with the testing rule will produce the same result on moving the tracer forward or backward.

- 4) Whether the turning u of the roller in tracing the same figure on the left and on the right of the base is equal, i. e. whether the axle of the measuring roller is parallel to the tracer arm; this also can be ascertained by means of the testing rule.

It is only to be born in mind that the circular line described in either position must not be too close to the base XX, as otherwise the result would be affected by its proximity and the error in the readings would not merely indicate the error which originated from a wrong position of the axle of the measuring roller.

If the results are too small by $\frac{1}{n}$ of the area the tracer arm must be shortened by $\frac{1}{n}$ of its length and vice versa. The graduation of the tracer arm gives this length (distance between tracer arm and axis of rotation) for this purpose with sufficient accuracy.



Calculation of the length of the tracer arm and the settings by means of the graduations on it

In order to find by simple calculation settings which are not given in the table in the lid of the case or to ascertain the length of the tracer arm, the graduation on the tracer arm in conjunction with the vernier affords an excellent mean.

For this purpose the table in the case gives the values of the unit of the vernier also for full size (1:1) in square mm or square inches. Now let a be the longest and a^1 the shortest of the settings of the tracer arm which the table in the case contains and let f^0 and f^{01} be the corresponding values of the unit of the vernier in square mm: we wish to find the setting a_2 for the value of the area f^0_2 . If F represents the length of the tracer arm for the value of the area $f^0 - f^0_2$ we have the following equation:

$$\frac{a - a^1}{F} = \frac{f^0 - f^{01}}{f^0 - f^0_2} \text{ from which we obtain } F = \frac{(a - a^1) f^0 - f^0_2}{f^0 - f^0_2} \quad 1)$$

$$\text{and } a_2 = a - F - \quad 2)$$

For instance, let a be 320,9, $f^0 = 10$ square mm, $a^1 = 128,5$, let f^{01} be 4 square mm; we wish to find the setting a_2 for the scale 1:2500 with 20 square mm as the value of the unit of the vernier: $f^0 = 3,2$ square mm.

According to the equation 1:

$$F = \frac{(320,9 - 128,5) (10 - 3,2) - 218,05}{10 - 4}$$

consequently the desired setting a_2 according to equation (2) is

$$320,9 - 218,05 = 102,85.$$

Before accepting the settings thus calculated as definite we convince ourselves of their correctness by tracing trial areas.

In the same way the length of the tracer arm (distant between the point of the tracer and axis of rotation of the pole arm) corresponding to the unit of area (10 square mm) can be ascertained and by simple proportion the length can be calculated for any other desired unit. In the above example the length of the tracer arm for 10 square mm is computed as 320,65, the vernier of the tracer arm is consequently displaced by $0 + 0,25$ on the existing length of the tracer arm, this constant value is also to be added to the computed length of the tracer arm, if negative, to be subtracted therefrom in order to find the correct number of the setting.

In order to find, for instance in the above case, the number of the setting for 6,4 square mm, we have simply to multiply by 0,64 the length of the tracer arm for 10 square mm, viz. $320,65 = 205,21$ and add therefore the above constant displacement of the vernier $+ 0,25$, we then arrive at the correct number of the setting, viz. 205,46.

If the length of the tracer arm is known, we can, if shrunk plans are to be measured, vary it in such a way that the planimeter indicates the correct superficial area. If, for instance, a plan is to be computed which has shrunk 1 % in one direction and 0,5 % in the other, the areas will be 1,495 % too small. We then have simply to shorten the tracer arm to 316,1 instead of 320,9 and the planimeter will give the correct superficial area.

$$(320,65 \times 0,015 = 4,81, 320,9 - 4,8 = 316,1)$$



Some Particulars to be retained in mind

The Rolling Sphere Planimeter requires a very careful handling, being the planimeter of high precision par excellence. Its mathematically true worked organisme: the cylinder and the spherical segment are to be kept absolutely free from dust which in the most cases is concentrating in the middle of the segment, where the largest number of revolutions are effectuated by the sphere and the cylinder. No true indications can result from this delicate organisme once by adherent dust deviated from its very mathematical conditions.

Protect your instrument against jar and pressure in order to maintain the perfect movement of the measuring roller.

By shifting sideways the planimeter on the paper a gradual smoothening of the rough surface of the roller R' will be produced and the accuracy of the roller motion impaired.

Never turn the measuring roller with the hand when it is working on its working surface.

The movement of the roller being affected by temperature and in consequence of the unequal expansion of the frame (being of brass) and the roller axle (being of steel) the roller will have a slow heavy play if the instrument has been kept in a cool place; in the contrary if it has been exposed to heat or to the direct sun rays the roller axle will have too much play and in both cases perceptible errors will result. In the former case it will be sufficient to warm somewhat the roller frame with the hand, in the other case the instrument should be allowed to cool before use.

On account of these facts it should be avoided to use the instrument without prealably be sure to have softened the direct effect of an extreme temperature.

The general variations produced in the dimensions of the instrument by change of temperature do not to the extend frequently imagined affect the measurement of the area.

The expansion of the tracer arm amounts for instance between 0° and 100° C to only about $\frac{1}{500}$ th. of its length; as however the temperature of a room varies, as a rule between 10° and 30° only, this would cause a variation of the arm and consequently of the turning u of the roller of $\frac{1}{2000}$ of its value at most.

It is indispensable to have the sheet of the paper, containing the figure to be measured, placed on a level, well planed and horizontal board. If this paper is of such a nature that it lies perfectly flat on the board it need not to be fixed and can be placed together with the instrument resting thereon in a suitable position for the tracing and in a good light.

