THE

Coradi Planimeters

DESCRIPTION AND INSTRUCTIONS

FOR THE

USE AND TESTING

WITH

A GENERAL ELEMENTARY EXPLANATION OF THEIR OPERATION

BY

G. CORADI, ZURICH.

1903

ZURICH
Printed by Aschmann & Scheller, Predigerplatz
1903

PRICES OF THE PLANIMETERS HEREIN DESCRIBED

(The Nos. are those of the catalogue)

No. 29	Small rolling sphere planimeter	Fig. 17, page 18
No. 30	Ditto with extension	" " " "
No. 31	Large rolling sphere planimeter	" " " "
No. 32	Ditto with extension	" " " "
No. 33	Precision disc planimeter...	Fig. 18, page 20
No. 34	Ditto with determination of the constant for pole within the figure	Fig. 18, page 20
No. 35	Compensation planimeter for one value only of the vernier unit	Fig. 23, page 24
No. 36	Ditto with graduated tracer arm and micrometer adjustment for 4 to 5 values of the vernier unit	Fig. 22, page 24
No. 37	Ditto with device for easy parallel adjustment of roller axle	Fig. 21, page 23

For all planimeters adjusted for two measures (english and metric) and testing rule with two measures, the prices are higher.

Precision pantographs *to*

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* The chapters marked with a star appear in **W. Caville's, Lehr- und Handbuch der Landmesskunst, published by Ernst, Halberstadt and Leipsic,** which book also gives historical notes on the invention of the planimeter and descriptions of planimeters of old construction.

PREFACE.

In compliance with oft repeated wishes I now present to the public the English translation of my treatise on planimeters, in the hope that this work will promote the use of these instruments which are so valuable for saving mental effort and time in the exact measurement of areas.

I take advantage of this opportunity to express publicly my indebtedness to Professor O. HENRICI for editorial services which he has kindly rendered and my thanks to Mr. Frederick Lucas, of 27, Leadenhall Street, London for his careful and precise translation from the German.

ZURICH, February 1903.

G. Coradi.

I. Introduction.

Accuracy in determining areas of plots of land depends on the exactitude with which lengths are measured. For land of great value, areas are generally computed from the original from the numbers obtained at the surveys.

I have been assured by experienced surveyors that with accurate plans on a sufficiently large scale areas computed by means of my planimeters are as accurate as those calculated from original figures, provided the instrument is correctly handled.

The word planimeter means, in general, any instrument used to calculate by mechanical means, directly from the drawing, distances and areas. The planimeters which we will describe give the areas in square metres or square feet. These instruments are called circumscribing planimeters, since by going round the perimeter of a closed figure with a tracer a number can be read off, which expresses the area enclosed.

Such a planimeter consists of at least two principal parts:

a) A horizontal bar *) one end of which carries the vertical tracer, whilst the other end, a vertical axis, is constrained, to move along a line called the guide-line. This guide-line may be any curve, but in practice the circle — for polar planimeters — and the straight line — for linear or rolling planimeters only are used. **)

b) A roller turning freely on its axis, and connected to the arm so that the axis of the roller is horizontal and parallel to the imaginary vertical plane through the axis of rotation of the arm and the point of the tracer. This roller is graduated to show the amount of lateral displacement of the arm during the travel; the displacement multiplied by the length of the arm gives the area of the figure contained.

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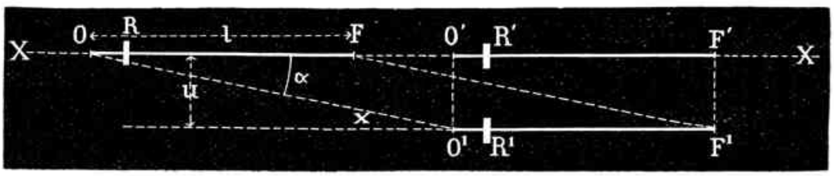
*) It is assumed in the following that the planimeter is only used on a horizontal plane, as is absolutely necessary for correct results.

**) In the tractory planimeter of Professor L. Kleritj of Belgrade, first made in August 1893, as also in the „hatchet" planimeter of Captain H. Prytz, constructed on the same principle, the guide-line is a curve (tractory) the form of which varies according to the figure circumscribed. Instead of the vertical axis of rotation the arm carries a roller with a knife-edge (in the Prytz planimeter, a hatchet shaped edge), the vertical plane whereof, if extended, passes through the point of the tracer. This edge (or edged roller) at the same time indicates the amount of lateral displacement of the tracer arm. Whilst the tracer is taken round the figure, beginning and ending at its centre of gravity, the cutting edge describes a curve on the paper. The distance between the initial and final point of the curve multiplied by the length of the tracer arm gives the area of the figure circumscribed. As this instrument does not possess sufficient accuracy for geodetic purposes and requires more time for computing areas than planimeters having a graduated roller, we may dispense with any further description.

II. General theory of Planimeters.*)

An area is generated by a moving line provided that the motion is not in the direction of the line itself. If a finite line moves parallel to its initial direction the area passed through is equal to the product of the length l of the line and the rectangular distance u, between the initial and final positions. (Fig. 1.)

Fig. 1



Let this finite line be straight and represented by a rod or „arm" O F. At its one end is the tracer F, at the other the vertical axis O. Connected with this axis and resting on the drawing is a roller R, the axis of which horizontal and parallel to O F and so arranged that the roller can turn on it without frictional resistance. If this arm is moved in the direction of its length O F as far as O' F', the bar does not generate any area, the roller R, the axis of which remains during the movement of its point of contact constantly parallel to itself, will not turn round its axis, but only slip.

We will call this position of the arm its initial or normal position, and the line described by the tracer in this position the base. Hereafter the base will be designated by XX.

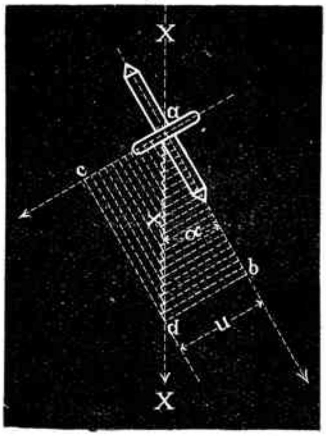
If now the arm is moved from its position O' F' and parallel thereto until attaining the position O^1 F^1 so that the arm describes a rectangle, the roller moves at a right angle to its axis and consequently rolls through an arc u which is equal to the rectangular distance between the two positions of the bar. The bar has then swept over the area $O' F' O_1 F_1 = O F O_1 F_1$ which is given by $J = l\ u$, i. e the area swept over by the arm equals the length l of the arm multiplied by the rectangular distance of the lateral displacement of the arm, or, (if XX designates the normal position) multiplied by the rectangular distance of the arm from its normal position.

If the arm is moved directly from the position O F to the position O_1 F_1 the movement of the roller can be considered as consisting of a number of infinitely short slipping movements in the direction XX and an equally number of infinitely short rolling movements rectangular to the axis of the roller, the sum of which $= u$, the slipping movements involving no turning of the roller. If the bar 0_1 F_1 is brought back from this position into its initial position O F either directly or through O' F', the roller turns to the same extent in the opposite direction, consequently no area has been described, since the two motions cancel each other. As all parallelograms

*) This description follows more or less that of Professor Stambach: The Coradi planimeters, their theory, construction and accuracy Stuttgart, 1889. See also W. Caville, Handbuch der Vermessungskunde.

on equal bases and of equal altitudes are equal in area, the bar could also be brought back from the position O_1 F_1 by another route to the base XX and the areas described, first positively and then negatively, would still cancel each other.

Fig. 2



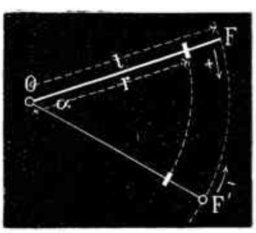
If the point of contact of the roller is moved from d to a (Fig. 2), i. e in a direction oblique to its axis the movement can be considered as consisting of infinitely short movements perpendicular to the axis and parallel to it; the former cause the roller to turn and when a is reached their sum equals the distance $ac = bd = u$. The latter movements only cause a slipping of the roller, the sum of which at a is equal to the distance $a\ b$. The turning u of the roller is equal to the distance x traversed by the point of contact multiplied by the sine of the angle formed by the axis of the roller with the direction of motion of the point of contact.

If the angle $b\ a\ d = \alpha$, then $u = x.\ \sin\ \alpha$ (1)

therefore the area described is $J = lu = l.\ x.\ \sin\ \alpha$ (2)

If the arm O F is turned round its axis O so that the tracer describes an arc F F_1 the arm turns through a sector with the angle α at the centre (Fig. 3).

Fig. 3.



With a distance r of the roller from the centre O the turning of the roller is

$$u = r\,\alpha \text{ consequently } \alpha = \frac{u}{r} \qquad (3)$$

and the area turned through $J = \frac{l^2 \alpha}{2} = \frac{l^2 u}{2r}$ (4)

For a whole revolution the ratio

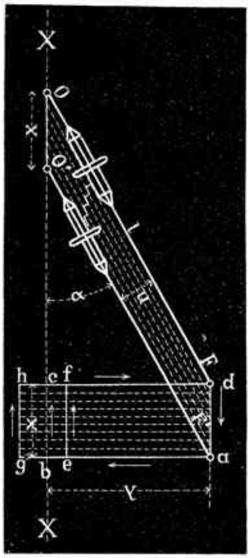
$$\frac{u}{r} = 2\,\pi, \text{ therefore } u = 2\,r\,\pi \text{ and } J = l^2\,\pi. \qquad (5)$$

If however the arm returns to its initial position without having completed a whole revolution the sector is described negatively and both $u =$ zero and J = zero.

It will now be clear from Fig. 4 that if the tracer F is moved from d to a the roller will turn through the magnitude u. This magnitude multiplied by the length l of the arm gives the area of the figure $abcd$ travelled around by F. The motions of the tracer from a to b and from c to d produce equal, but opposite turnings of the roller, whose sum is zero. It can also easily be seen from Fig. 4 that it is immaterial to what part of the arm the roller is fixed, it can also be fixed to the side of the arm provided the axis of the roller is parallel to the arm and parallel to the plane worked on. If the tracer travels from b to c, i. e along the base, the roller only slips without turning on its axis. A permanent turning u of the roller is therefore only produced on travelling along $d\ a$, which turning is equal to the altitude of a rectangle, the base of which is the tracer arm O F $= l$.

From this we can deduce the following rule for the signs of the area:

Fig. 4



On the right of the base the direction of the motion of the tracer is positive
On the left of the base the direction of the motion of the tracer is negative
In the direction O to F of the arm the turning of the roller is positive
In the direction F to O of the arm the turning of the roller is negative (7)

If therefore the tracer travels on ef instead of on the base, the motion of the tracer is positive and the turning of the roller from e to f negative, therefore the area $ebcf$ is subtracted from the area $abcd$, because $(+l).(-u) = -lu$. If the tracer arm crosses the base, the motion of the tracer is negative, but the turning of the roller from g to h is also negative, the roller therefore adds the area $bghc$ to $abcd$ and the turning of the roller is proportional to the whole area $aghd$ travelled around,

because $(-l).(-u) = +lu.$

If we imagine the area $aghd$ to be an infinitely narrow rectangle we can consider any figure to consist of such infinitesimal rectangles.

If the guide line and consequently also the base is a circle (polar planimeter) we consider the elements of area to be infinitely short trapezoids bounded by radii and arcs.

As the tracer moves along the two radii the arm O F describes the same area once positively and once negatively. As it moves along the base, O F describes the elements of area lying between the base and the guide line in a negative direction, but these elements are again described positively whilst the tracer moves along an arc outside the base, and consequently their sum is again zero, so that on moving the tracer along an arc outside the base there only remains the area bounded by the arc, the two radii and the base described in the positive sense. If the tracer travels round an element of area lying within the base, O F moves negatively over an element between the basecircle and the guide-circle only in so far as it is not contained within the element travelled round. The process is consequently in its final result exactly the same, only somewhat more complicated, than for a straight line as a guide-line.

The area traced round is therefore always equal to

(a) $J = l.\ u =$ a rectangle whose base l is the length of the tracer arm O F and whose altitude u is the measure of the lateral displacement of the arm or, according to Fig. 4.

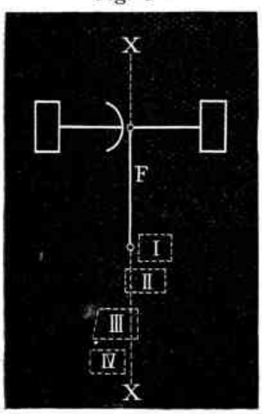
(b) $J = y.\ x = l \sin \alpha.\ x =$ the rectangular distance of the tracer from the normal position of the tracer arm multiplied by the forward movement x of the turning point O on the guide-line.

The measure of the lateral displacement of the tracer arm O F is given by the turning u of the roller. If the roller is allowed to rest on the plane of the plan, $u = x . \sin \boldsymbol{\alpha}$. If the roller is caused to turn on a disc or sphere, the revolutions of which are always proportional to the movement x of the point O on the guide-line, then the distance, traversed by the roller on the plan i. e. x, is increased so that the roller can measure smaller strips of area. For the rest, the truth of the equation (2) is unaffected, it assumes the general form

$$J = l. u. c = l. \sin a. x. c.$$

in which c expresses a constant depending on the dimensions of the instrument.

Fig. 5



If the tracer is taken along the base XX, with the tracer arm in its normal position, the axis of the roller remains constantly parallel to the direction of motion of its point of contact and the roller will only slip, not roll. The line, which the point of contact of the roller describes whilst the tracer travels along the base, can therefore be called the slipping line. If the guide line is a straight line, as with the rolling planimeter, the base and the guide lines coincide or are parallel straight lines. Fig. 5. If the roller turns on a sphere, the slipping line is at the pole of the sphere, and consequently forms only a point, or a small circle the radius of which equals the vertical distance between the axis of the sphere and the axis of the roller.

With the polar planimeter Fig. 6, the guide line is a circle. In this case the normal position of the arm F is that position in which the extended plane of the roller passes through the pole P, i. e through the pivot of the whole instrument. If the arm is turned in this normal position round the pole, the

Fig. 6

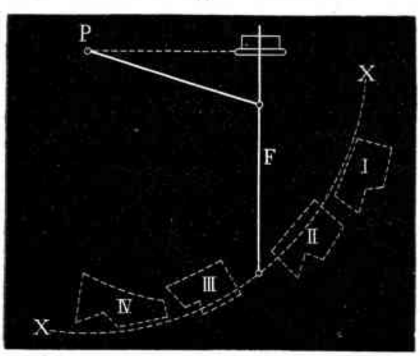
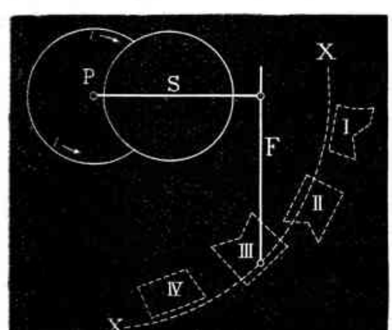


Fig. 7



tracer describes the base XX, the point of contact of the roller describes the slipping line and the axis round which the arm turns describes the guide-line. These lines are three parallel arcs, having their common centre at P.

If the roller does not turn on the plan itself, but, as in the case of the disc planimeter, on a revolving disc (see Fig. 7) placed on the pole-arm, the position of the tracer arm rectangular to the pole-arm is the normal position of the planimeter; the base is a circle described round the pole in this position of the tracer, and the slipping line is a circle on the disc, its radius being equal to the distance of the point of contact of the roller from the centre of the disc through which the extension of the plane of the roller passes.

It will be seen that in all kinds of planimeters a slipping of the measuring roller occurs on travelling along the base. This slipping movement is greatest with the polar planimeter, the roller of which turns directly on the plan; it is the smallest with the sphere planimeter, but even here it is by no means absolutely excluded.

This slipping of the roller has always been considered as the main source of error in planimeters. But the general opinion seems to have been that the errors were to be accounted for by roller performing small rollings, when passing along the slipping line, instead of merely slipping without rolling at all.

Experiments extending over many years have led me to form a different opinion, namely, that if the tracer is moved **close** to the base and parallel thereto the roller only slips **without** turning, and that this fact is due to the frictional resistance which the axle of the measuring roller encounters in its bearings. This friction of the axle can however, in the disc and compensation planimeters, be remedied by the so-called „milling" (innumerable lines, parallel to the axis of the roller, with which the whole edge of the roller is uniformly covered) made on the circumference of the measuring roller so that in new instruments with carefully balanced, freely moving axles of the measuring rollers, the base exerts no injurious influence on the results.

This condition however, can only be permanently maintained with the most careful handling of the instrument; besides care must be taken so to place the planimeter that the base does not injuriously affect the result. The following rules should therefore be well borne in mind.

III. General rules for all planimeters with measuring roller.

1) In every circumscribing planimeter whatever be its name, the axle of the measuring roller must have very fine bearings and move very freely, with as little friction as possible.

2) In all planimeters the axle of the roller must be treated with the utmost care and protected against jar and pressure, if the instrument is to maintain its accuracy. If the fine points of the axle of the roller are injured, the roller can no longer move in perfect accordance with the law $u = x \sin \alpha$ and the accuracy of the planimeter can only be re-established by repairs.

3) The instrument must always be placed in such a position with respect to the figure to be measured that its boundaries do not pass close to and parallel to the base.

In Fig. 8—11 the favourable positions of the area J and the instrument are indicated for the different kinds of planimeters. The favourable position can be ascertained without lengthy experiment by the following rule:

Fig. 8

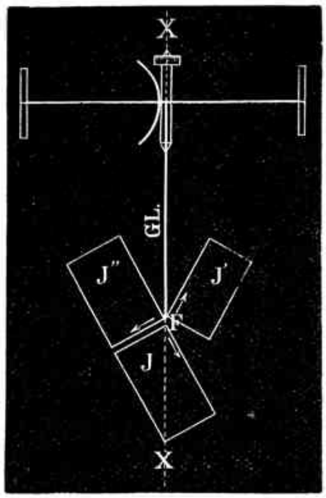


Fig. 9

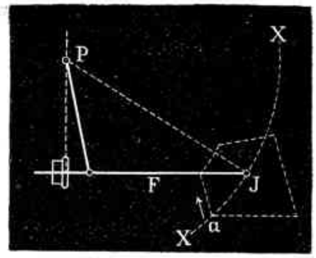


Fig. 10

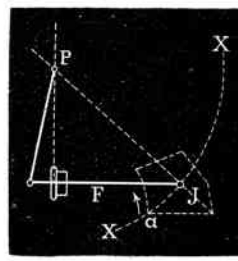
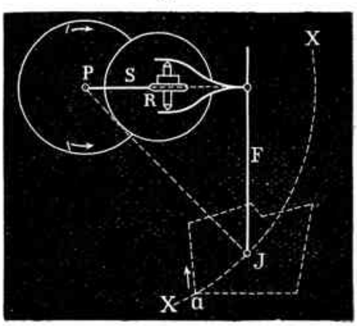


Fig. 11



4) Place the tracer approximately in the centre of the area to be measured and place the pole P so that the plane of the roller, if extended, passes through the pole; with the rolling planimeter so that the tracer arm and the roller are at a right angle to each other. With this mutual position (i. e. on the base) move the tracer to the boundary of the figure at α and start the motion of the tracer from this point, because on the base the roller makes the least movements and consequently errors in returning the tracer to the initial point will be the more easily avoided.

Fig. 5, 6, 7, shew unfavourable positions of the instrument which are to be avoided. By always observing rules 2, 3 and 4 and making certain that the planimeter complies with rule 1, the instrument will always furnish reliable results.

It is universally admitted that the nature of the surface under the roller will influence the turning of the roller. On smooth flat paper the turning will not be the same as on material having a rough, granular, fibrous or, worst of all, uneven and undulating surface.

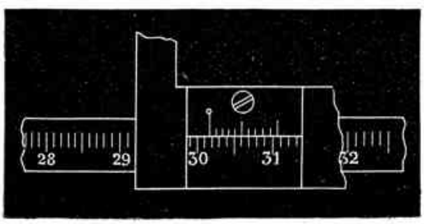
The possibility of preserving the surface on which the roller works and of reading small areas constitutes the undoubted advantage of the disc and sphere planimeters and for this reason these planimeters are well worthy of especially careful treatment in addition to that which they require owing to their construction, as even the most simple planimeters require careful handling if they are to give reliable results.

IV. Parts which are alike or similar in all planimeters.

1) The tracer arm.

The tracer arm is a hollow rectangular bar of nickelled brass. It is adjustable in a sleeve carrying the vertical axis of rotation, in order to be able to vary its length l. The graduation in $^1/_2$ millimetres is not for measuring areas, but only for the purpose of determining accurately to $^1/_{20}$ mm, by means of the vernier at the sleeve, the relative length l of the bar in order that a revolution of the roller may correspond to a prescribed unit of area. The sleeve is made with a silver plated bevelled edge to facilitate the adjustment of the bar to special marks for certain scales and values of the vernier unit. The exact adjustment is done by means of the vernier and the graduation. Fig. 12 gives as „setting of the vernier on the tracer arm" the figure 301,5. For the accurate setting to the figures given in a table in the lid of the case the micrometer screw is used which is fixed in a shorter sleeve embracing the tracer arm: first the clamping screw of this sleeve is tightened and then the micrometer screw turned until the vernier reads the required figure, and then the clamping screws of the sleeve on the tracer arm are tightened.

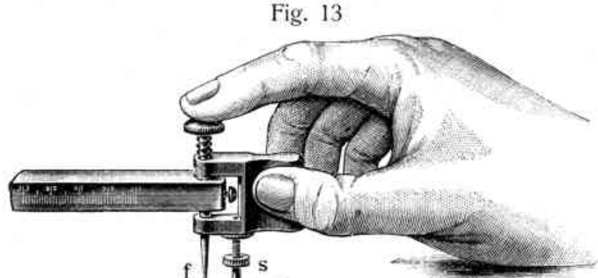
Fig. 12



2) The tracer with its handle and support.

The tracer of all my planimeters is now made in the way shewn in the illustration Fig. 13. A hardened steel pin pointed at its lower end is vertically

Fig. 13



fixed by a set screw in the right end of the tracer arm. A handle b which turns easily about the vertical shaft of the tracer carries a support s consisting of a pin with rounded foot between the handle b and the tracer arm there is a spiral spring to keep the point of the tracer just clear of the paper. The handle is grasped with the **thumb** and **middle** finger; the index finger is thus free to press the knob of the tracer into the paper in order to mark the starting point accurately or to keep the tracer fixed whilst the hand slides to a new position on the paper (see fig. 13).

When **using the testing rule, the support s must be screwed up,** for which purpose it is provided with a milled head and lock nut.

3) The measuring roller and counting wheel and their divisions.

The measuring roller is the most delicate part in all planimeters. The two finely made axle pivots are of the best hardened steel. They work in extremely fine centres in cylindrical steel pins. With the polar planimeter and disc planimeter the axle carries a circular disc of nickel steel with a curved rim, the turning u of the roller being dependent on the diameter of this disc. Nickel steel is chosen, because it is not affected by rust as is ordinary steel, in which case the rust would destroy the fine milling on the rolling circumference and thereby affect the accuracy of the rolling.

With the sphere planimeter the diameter of the roller, which measures the rolling, is the diameter of a cylinder the generatrix of which is a mathematically true straight line parallel to the tracer arm. This line when touching the sphere determines the radius of the circle of contact between the sphere and the cylinder; this radius increases proportionally to $l \sin \alpha$, so that the turning u of the cylinder equals $J = C.\,xl.\sin \alpha$. As the cylinder only rolls on the sphere and does not slip, the cylinder need not be milled but must be kept with a clean metallic surface. The axis of the cylinder is made of a white metal alloy of sufficient hardness which does not easily oxidize.

Each axle of the measuring roller carries a white celluloid cylinder with 100 divisions; very close to this cylinder, but not touching it, is placed a fixed cylindrical segment of the same material which carries the vernier which subdivides each of the 100 intervals of the main graduation into 10 parts, so that a thousandth part of a revolution of the roller can be read. The axle of the roller carries a worm, in which a counting wheel engages, which in ordinary polar planimeters is furnished with 10 teeth and with 50 teeth in the disc and rolling planimeters. The axle of the counting wheel carries a celluloid dial with the same number of divisions as the counting wheel has teeth; with each revolution of the roller the pointer on the dial advances one interval. Each interval on the dial therefore represents 1 revolution of the roller or 1000 units of the vernier.

Fig. 14

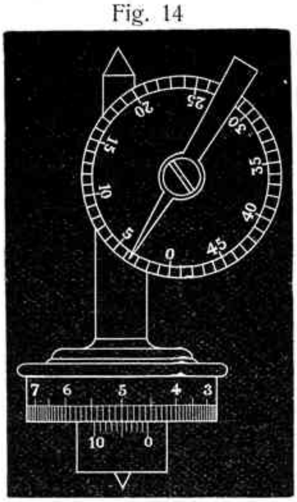


Fig. 14 illustrates the roller and dial of a disc planimeter and gives, for example, the following reading: the index on the dial stands between the 3rd and the 4th division consequently 3 full revolutions of the roller, starting from zero, have been performed.

The first figure of the reading is therefore	3000
The zero of the vernier stands between figure 4 and 5, consequently from the zero of the roller 4 full hundreds only have been turned; the second figure is therefore	400
Of the ten lines lying between 4 and 5 the fifth line has passed the zero point of the vernier:	
This gives the tenths as	50
The fifth line of the vernier exactly coincides with a division of the graduation, this gives the unit	5
The whole reading is therefore	3455

The cylindrical steel pins carrying the axle of the roller are in a brass frame and kept in place by steel set screws. Before the position of the pins can be altered, these set screws must be slightly loosened and after adjusting the pins they must be screwed up again. A steel screw parallel and close to the pin, engages with its flanged head in a notch of the steel pin and enables the pin to be adjusted in the direction of the axle of the roller.

This device enables a perfectly parallel adjustment of the roller axle, but owing to the lateral action the pin cannot be regulated without allowing for some back lash of the flanged screw which should be borne in mind, when adjusting the movement of the roller by shifting the position of the pins. Unless absolutely necessary no screwing should be done at the roller bearing. With careful handling the instrument will not vary for years. The roller is so adjusted that it turns freely at the usual temperature of the room, when not resting on the working surface.

No roller when resting on the working surface should ever be turned with the hand.

A change of temperature affects the movement of the roller by reason of the unequal expansion of the frame (brass) and the roller axle (steel). If the instrument has been kept in a cool place the roller will have a slow heavy movement (which according to chapter II affects the turning of the roller). If the instrument has been exposed to heat, for instance, to the rays of the sun, the roller axle will have too much play, which likewise leads to errors.

In either case it would, however, be wrong to attempt to adjust the movement by altering the position of the pins. In the former case the frame of the roller should be somewhat warmed with the hand, in the latter case the instrument should be allowed to cool in the shade before use. On account of this effect of the temperature, the window should in cold weather not be left open during the use of the instrument.

The other variations produced in the dimensions of the instrument by change of temperature do not affect the measurement of areas to the extent frequently imagined. The expansion of the tracer arm amounts for instance, between 0^0 and 100^0 C, to only about $^1/_{500}$th of its length; as however, the temperature of the room varies, as a rule between 10^0 and 30^0 only, this would cause a variation of the arm and consequently of the turning u of the roller of $^1/_{2500}$ of its value at most.

4) The constant if the pole is within the figure.

If larger areas are to be measured, those planimeters of which the guide-line is a circle (i e. polar planimeters) can be used with the pole placed inside the figure; the tracer is then as before moved round clock-wise. If in this case the tracer travels along the base XX and back to its starting point no turning of the roller has taken place, since it moves on the slipping line; in this case the area enclosed by the path of the tracer equals $r^2 \pi$ ($r =$ distance of the tracer from the pole) and consequently equals the area bounded by the base XX. As in this case no turning of the roller takes place, we substitute for it a constant which represents the turning of the roller corresponding to this area in units of the vernier. This constant differs for each instrument and each length of the tracer arm and is in each separate case determined by trial.

If the area traced is greater than the base area, the turning of the roller obtained must be added to the constant; if the former area is smaller than the latter, the turning of the roller must be substracted from the constant.

As this way of using the instrument is somewhat complicated and its accuracy not very great it is seldom adopted.

By using the latest construction of the adjustable pole-arm in compensation planimeters the application with the pole within the figure is considerably simplified, and besides, going twice round the contour of the figure, once with the pole to the right of the tracer arm and once with the pole to the left, a sufficient accuracy also is obtained for this rare application of the planimeter.

Fig. 15

In the above figure 15 the adjustable pole-arm is shewn in about half size. It consists of two principal parts, the rectangular sleeve H to which the weight b is attached, and of the adjustable arm P, which carries the joint ball D and can be adjusted as desired in the sleeve H and fixed by a clamping screw d so that the distance between the pole pin b and the joint ball D can be varied between 13 cm and 23 cm. The arm P possesses a short graduation in $^1/_2$ mms on which the lenght of the pole arm is adjusted by means of the bevelled and silvered face on the sleeve H. The length of the pole-arm can be so chosen that the base circle equals an area of 20,000 units of the vernier. During the tracing of an area with the pole inside the measuring roller adds to 20,000 as many units of the vernier as the area is greater than 20,000 (which is the base circle) or subtracts as many units of the vernier from 20,000 as the area is smaller than the base circle: the result obtained represents therefore at once the superficial area of the figure in the same way as with the pole outside the figure; but care must be taken to see which of the figures 1 or 2 gives the ten thousands of the readings. This can easily be recognised. The length of the pole-arm must be determined for each individual adjustment of the tracer arm and is marked by divisions on the adjustable piece; besides, the position of the bevelled edge on the scale of the arm is marked in the table in the lid of the case, instead of the „constant".

5) The table in the lid of the case.

With each planimeter a table of the following form is supplied fixed in the case, which table contains the lengths of the tracer arm separately determined for each instrument, the values of the units of the vernier (and of the constants for polar planimeters) for 4 or 5 different scales.

Scales	Position of the vernier on the tracer bar	Value of the unit of the vernier on the measuring roller		Constant
			(1 : 1)	
1" = 50'	343,8	40 □'	0,016 □"	23138
1" = 25'	"	10 "	" "	"
1" = 20'	322,3	6 "	0,015 "	23309
1" = 100'	"	150 "	" "	"
1" = 40'	268,7	20 "	0,0125 "	24241
1" = 20'	"	5 "	" "	"
1" = 30'	238,8	10 "	0,0111 "	—
1" = 60'	"	40 "	" "	—
1" = 100'	215,0	100 "	0,01 "	—
1" = 200'	107,1	200 "	0,005 "	—

For metric measure

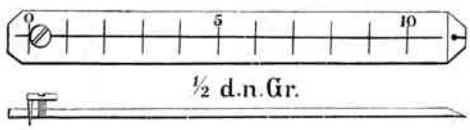
			(1 : 1)	
1 : 1000	333,3	10 qm	10 qmm	22173
1 : 500	266,6	2 "	8 "	23154
1 : 2500	213,3	40 "	6.4 "	—
1 : 2000	166,6	20 "	5 "	—
1 : 5000	133,3	100 "	4 "	—

The first vertical column contains the scale to which the plan of the area is drawn. In the second column the length of the tracer arm is given in divisions of the vernier on the tracer arm: the third column contains the value of an unit of the vernier of the roller, or the factor by which the turning u of the roller, obtained in such units of the vernier by going once round the contour of the area, must be multiplied in order to obtain the area expressed in square feet. In the same column the values of the unit of the vernier are given in square inches on the scale 1 : 1. The fourth column contains the constant for using the instrument with the pole within the figure. For planimeters having an adjustable pole-arm the 4th column gives, instead of the constant, the length of the pole-arm for the constant 20,000 in ½ mms.

6) The testing rule.

We supply with each planimeter a small rule (Fig. 15) called the testing rule which is divided in 8 cm or 10 cm or for English measure in 3" or 4".

Fig. 16.



At the zero of the graduation a small hole is bored through which a needle point passes, kept in place by an overlapping screw. Each line of the graduation has a small conical hole, in which the point of the tracer can be placed. The needle point is pressed into the paper so that the rule lies flat on the paper. If we now place the point of the tracer in one of the holes (after screwing up the support s) and turn the rule round the point in the centre, the tracer describes a circle of known radius. The bevelled end of the rule has an index line. This is set to the starting point which is to be marked on the paper by a pencil line. From figures 25, 26 and 19, page 33, the application of the testing rule can be seen: it is used for testing the uniformity of the turning *u* of the roller.

In order to keep the latter perfectly free from the pressure of the hand guiding the instrument (radial pressure of the hand) the tracer arm has to be loaded close to the tracer with a weigth of lead or the like and the testing rule is guided with the hand instead of the knob of the tracer.

The testing rule can be replaced by a testing disc, which is a brass plate about $1^1/_2$ mm thick. It is pressed against the plan and kept thereon in an immovable position by two steel points projecting from its lower surface. In its surface ground perfectly flat circles of 1" 2" and $2^1/_2$" radius are engraved, in which the point of the tracer can move. It is not necessary to mark the starting point of fhe tracer, if it is chosen in that position where the plane of the tracer arm, or its extension, passes through the centre of the circle, since at this place the roller changes its sense of turning.

After having thus described the individual parts which are alike or very similar in all planimeters we come now to the description of the instruments themselves.

V. Description of the planimeters.

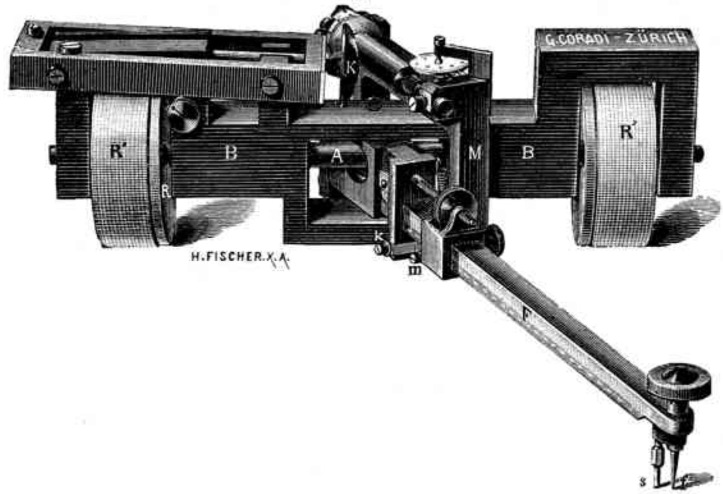
1) The Rolling Sphere Planimeter.

Fig. 17 represents the instrument in about half size.

The guide line of the pivot of the tracer arm F is, as with all linear planimeters, a straight line. The instrument rests on the plan at three points, the two rollers R' and the tracer F or its support s. In the frame B the axle A with fine hardened steel pivots works in two centre screws which have their threads in the frame B. The right hand screw can be adjusted and, like all adjusting screws in my planimeters, can be fixed by a brass set screw.

The two cylindrical rollers R' are rigidly connected with the axle A: they are of equal diameter, concentric to the axle A and provided on their circumference with a kind of dotted milling in order to increase their parallel motion on the plan.

Fig. 17.



On the face of one of these rollers is a wheel with fine teeth. In it gears a small toothed wheel (not shewn in the drawing) which is fixed on the steel axle of the spherical segment K. This axle is supported in an horizontal frame, on the left by a highly finished hardened pivot in an adjustable centre screw and on the other side by a hardened cone in a hardened steel plate. Outside this plate on a cone of the axle a spherical segment K is fixed its axis being coincident with that of the axle. It is made of a hard metal alloy not liable to oxidation and carefully ground into its proper shape. The left part of the frame of the axle and consequently the sphere itself can be somewhat raised on turning about horizontal pivots engaging in the frame B. It falls by its own weight until the small wheel on the axle rests on the wheel of the cylindrical roller whereby the proper gearing is automatically secured. By a half turn of the screw marked with an arrow the small wheel can be disengaged; if the arrow of the screw points downwards, the small wheel again gears.

The axle A and the axle of the sphere are parallel and in the same vertical plane. In this plane, at the centre of the frame B, the vertical axle of rotation of the tracer arm is fixed; this axle consists of two adjustable steel screws fixed in the frame, the hardened points of which engage in two centres which are bored into the sleeve of the tracer arm in such a way that their line of connection (i. e. the axis of the tracer arm) is rectangular to the tracer arm and that the imaginary vertical plane passing through the axis of the tracer arm and the point of the tracer, is parallel to the tracer arm. The latter and its construction have already been described in chapter V section (I).

The set screw for fixing the tracer arm in its sleeve is on the back of the frame B. To make it easily accessible, the tracer arm is turned as far as possible to the right.

The frame M carrying the cylindrical measuring roller catches under the tracer arm at the front and back and can turn about a horizontal axis parallel to the tracer arm. The axle consists of two adjustable screws in the frame M the hardened points of which work in centres in the sleeve of the tracer arm. The centre point turned towards the tracer is inserted in a small steel plate laterally adjustable by the two screws k, in order to bring the measuring cylinder into a perfectly parallel position to the tracer arm.

A spiral spring suspended at the frame M on the one side and at the tracer arm sleeve on the other, draws the frame M up against the spherical segment k, so that the measuring roller is always in contact with the spherical segment.

A screw with a cylindrically milled head, in the frame M, which presses against the tracer arm enables the frame M to be moved gently away from the sphere, thus destroying the contact between the sphere and cylinder. This should always be done when the instrument is not in use or before it is handled in any way, so that any jar of the cylinder against the sphere and injury of these two delicate parts (sphere and cylinder) is avoided. For this purpose we can also place a cushion of silk paper or soft leather between the sphere and cylinder. On the right side of the frame B a small screw is fitted, which is easily turned and prevents movement of the roller A so that the instrument cannot be accidentally started and injured.

The tracer arm can make an angular motion of about 30^0 left and right of the base; the magnitude of the movement in the direction of the base is unlimited. This instrument can consequently in one operation measure areas of unlimited length and of a width equal to the length of the tracer arm used.

The rolling planimeters are made in two sizes. The larger size has a roller 16 cm in length, the smaller one of $12^1/_2$ cm. The whole length of the tracer arm is 30 cm for the larger instrument and 24 cm for the smaller. With both we can supply (but only when ordered with the instrument) an extension to be attached to the tracer arm of up to 50 cm in the former and 40 cm in the latter case. This extension is attached to the tracer arm by drawing the latter out of its sleeve. Before attaching and removing this extension a *cushion is placed between the sphere and cylinder or these parts are removed, by releasing the screw, so as to protect them from injury.* The value of the vernier unit can be varied with the large instrument, according to the adjusted length, between 0,0016 □" (1,0 sq mm) and 0,0005 □" (0,4 sq. mm) with the small instrument between 0,00125 □" (0.8 sq. mm) and 0,0004 □" (0.32 sq. mm). By the extension of the tracer arm these values can be increased to 0,003 (2 sq. mm) and 0,002 □" (1.5 sq. mm) respectively. The large rolling planimeter with 50 cm length of tracer arm and 0,003 □" (2 sq. mm) value of the vernier unit can consequently measure, by one guiding round of the tracer, areas of up to 20" (50 cm) width and any desired length, which cannot be done by any other planimeter.

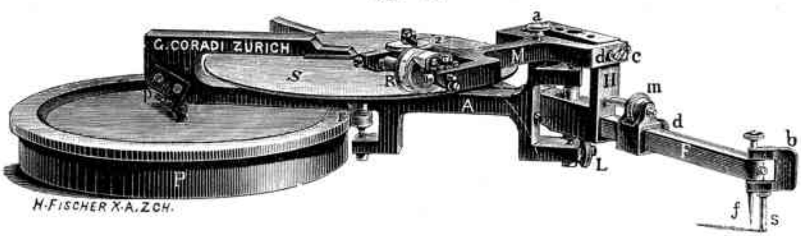
The rolling planimeter must not be shifted sideways on the paper, to avoid a gradual smoothening of the rough surface of the cylinder whereby the accuracy of the parallel motion would be impaired.

With the rolling sphere planimeter the measuring roller performs exclusively rolling movements on the surface of an accurately ground spherical segment; the result of the turning of the roller is therefore unaffected by the slipping or the condition of the paper whereon the figures to be measured are drawn.

2) The Precision Disc Planimeter.

Fig. 18 represents the instrument in about $^{1}/_{3}$ of its natural size. It consists of two separate parts, the pole-disc P with a finely toothed rim and the planimeter proper A F H S M.

Fig. 18.



F is the tracer arm of the usual construction, A the pole arm in which between pivots and on an adjustable centre screw, a vertical axle is placed which carries the small wheel r and the disc S. The two arms are connected by a vertical axle which is fixed in the sleeve H of the tracer arm and moves between hardened pointed screws, the top one of which is adjustable.

The pole arm rests on the plan by means of the roller L and is further supported by the bearing p on the finely polished steel ball which is fixed exactly at the centre of the disc P and forms the centre about which the whole instrument turns. The ball bearing p is a groove placed in a suitable oblique position and the semi-circular cross-section of which is equal to the diameter of the finely polished and hardened steel ball. If this bearing p is placed over the ball, the pole arm by its own weight sinks down on the sphere until the finely toothed small wheel r of the axle S of the disc comes in contact with the toothed rim of the pole-disc P, so that by the weight of the pole arm a continuous gearing of the small wheel is automatically effected and the whole instrument, resting on three points (the running roller L, the pole bearing p and the support s of the tracer) can be turned about p, whilst the small wheel r turns on the pole disc and thus causes the disc to turn proportionally to the movement of the axis of the tracer arm.

Over the disc S the frame M of the measuring roller described on page 13 is pivoted on a horizontal pointed axle parallel to the tracer arm and engaging in the sleeve H of the tracer arm. Of the two steel pins which form the axis of the frame the right hand pin has a larger diameter than the other and its pivot working in the centre in the sleeve of the tracer arm, is eccentric, so that by turning the pin the position of the frame M — and consequently the position of the axle of the roller — can be somewhat varied and the tracer arm and axis of the roller be placed perfectly parallel.

Both pins are kept in their places by steel set screws and are moved, in the same way as the pins of the axle of the measuring roller, by lateral flanged screws.

By its own weight the frame M descends until the roller rests on the disc S; by means of the screw *a* which rests against a plate fixed on the sleeve H, the frame M can be raised until the roller no longer rests on the disc S. The measuring roller should always be raised from the working surface when the instrument is not in use or before subjecting it to any manipulation (adjustment of the tracer bar, correction, cleaning etc.).

The frame M can be turned backward so that the disc can be easily cleaned.

The disc S is made of aluminium; its upper surface, covered with paper is smooth and proportional to its axis; its bottom surface is provided with radial ribs so that this disc is lighter than those made of ebonite or celluloid which in most cases become distorted (concave); it is lighter, tougher and stronger than those made of glass; aluminium discs are as durable as those of steel or brass which materials cannot be used owing to their excessive specific gravity and the consequent heavy movement of the instrument.

The pole disc P has a diameter of 15 cm; the total length of the tracer arm is 35 cm. The value of the vernier unit can be varied between 0,0003 □" (2 sq. mm) and 0,000075 □" (0.5 sq. mm) according to the adjusted length of the tracer arm. On the pole disc are two arrow marks which denote that part of the pole disc rim which produces the most uniform turning of the small wheel r. The instrument should always be used in such a way that the disc S moves within these arrows (see fig. 7 and 11).

If the tracer is in its normal position the plane of the measuring roller, if extended, passes through the centre of the disc; with this position of the tracer arm no turning of the roller is therefore produced by turning the instrument about the pole. If the tracer arm is turned about its vertical axis, the pole arm being stationary, the roller slips on the disc in an arc and, according to our former explanations, the turning of the tracer arm produces no turning of the roller. If the tracer arm forms an angle α with its normal position, the distance of the plane of the roller from the centre of the disc is proportional to $l.$ sine α. If with this position of the tracer arm its axis of rotation passes through the distance x (this being the *m*th part of the whole circumference of the guide line) therefore $\frac{2r\pi}{m}$; where r designates the radius of the guide-circle, the turning u of the roller will equal $u = l.\ sin\ \alpha\ \frac{2r\pi}{m} C$*), and be consequently always proportional to the area travelled round by the tracer.

This planimeter gives very accurate and reliable results and, as only two points (the supporting roller L and the tracer F) move on the plan it can be used for the accurate measurement of areas on rough uneven paper. But in all cases the pole disc must be well and securely placed, which can be done by the insertion of paper wedges if the working surface is uneven.

3) The Compensation Polar Planimeter.

These planimeters replace the old constructions of the simple polar planimeter. They differ from the latter especially in that their pole arm can be placed on both sides of the tracer arm. It is thus possible to eliminate the error in the turning of the roller u, due to the nonparallelism of the axis

*) C represents a constant dependent on the dimensions of the instrument (pole-disc P, small wheel r, pole-arm A). With regard to a strict theory of the disc planimeter see the essay by Prof F. Lorber in the „Zeitschrift für Vermessungswesen", 1884, No. 1.

of the roller and the tracer arm, by a double tracing, one with the pole on the left and one with the pole on the right of the tracer arm, as the error enters in the result once positively and once negatively. Surveyor O. Lang of Neuwied, on whose suggestion this improvement of the polar planimeter was made, has published the theoretical proof of the above principle in his paper in the „Zeitschrift für Vermessungswesen", 1894, No. 12.

If the axis of the measuring roller forms with the tracer arm the angle δ, the turning of the roller is no longer = x sine α, but u = x sine $\alpha \pm \delta$, therefore no longer proportional to the surfacae turned through by the tracer arm; according as the sign of the angle is + or −, the error in the turning of the roller will reach its positive or negative maximum and gradually decrease, as the pole arm and tracer arm approach the same straight line; now if the pole-arm can be still further turned so that its position changes over to the left side of the tracer arm the sign of the angle δ is reversed and the error still further decreased so that the turning u of the roller reaches a minimum in a position close to the pole and on the left of the tracer arm, if it formed a maximum in a position close to the pole and on the right of the tracer arm — and vice versa.

The arithmetic mean of two tracings of a figure, each with a symmetrical position of the pole on the left and right of the tracer arm, will therefore yield a result completely freed from the error due to the position of the axis of the roller. Two symmetrical positions of the tracer arm with respect to the figure, with the pole on the right and left respectively of the tracer arm, will at once be obtained if the pole, fixed according to rule 4, Chap. III, is left in its place and the tracer arm, after the first tracing, is brought into the same line as, and pushed underneath, the pole arm to the opposite side, as the telescope of a theodolite.

Fig. 19 shows two symmetrical positions of the tracer arm and the figure with the pole on the left and right of the tracer arm from the same pole position P. Fig. 20 represents two symmetrical positions of the area J and the tracer arm F trom **two** pole positions P and P". The error can be eliminated in either way.

Fig. 19

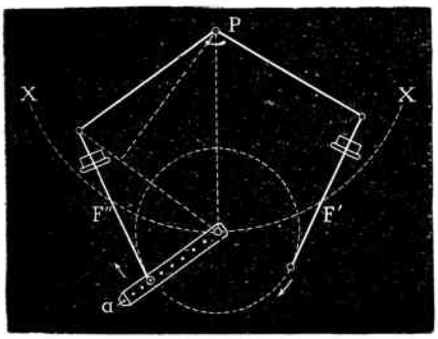


Fig. 20

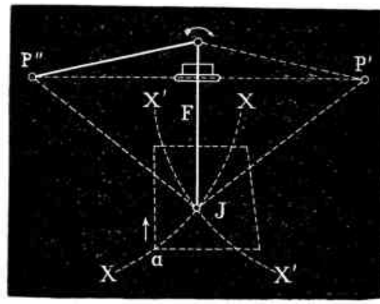
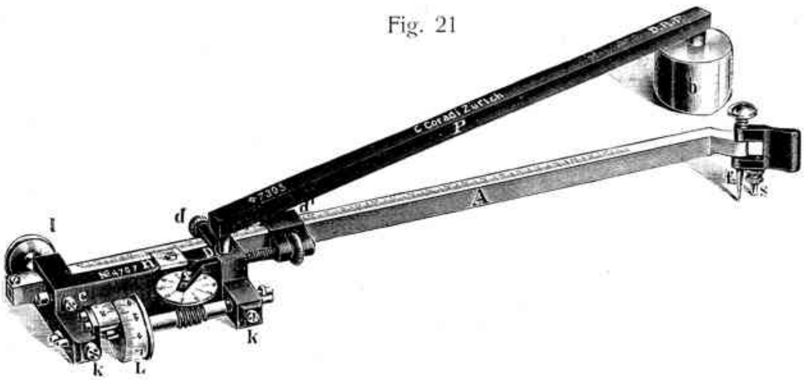


Fig. 21



In Fig. 21 the compensation planimeter is represented in about $^2/_5$ of its full size. It consists of two parts, the tracing frame A L l and the pole arm P, which can be put in the case separately. The tracing frame rests independently on three points: the measuring roller L, the tracer *f* or its support *s* and the roller *l*. The connection between the pole arm and the tracer arm is effected by a ball and socket joint, placed in the tracer arm as close as possible to the plane of the plan. The finely polished steel ball is fixed in the pole arm and in order to form the connection, is simply placed in the opening D in the tracer arm. This ball and socket joint forms the axis of rotation of the tracer arm which by means of the pole arm moves on a circle as guide-line; at the same time it enables the tracing frame always to rest with its three points on the plan, even if this is not flat.

The pole consists of a brass cylinder *b* screwed in at the right end of the pole arm. Its lower surface is shaped like a gable forming an edge at right angles to the pole arm which by turning about this edge can be lowered until its left end which carries the ball is firmly secured in the socket at D. In the centre of the cylinder ***b*** a small steel pin is inserted, kept in place by a set screw at the side. This pin terminates on both sides in a fine hardened point, which slightly projects under the lower edge of the cylinder ***b***. This point must not be pressed into the paper, but only be placed loosely on it; the instrument will thus be better secured and the plan not be spoilt by pin pricks. When the point has become blunt the pin can be reversed (the upper end be turned downward) and finally both points can be reground, or renewed.

If the pole-arm be slightly inclined sideways, without lifting it out of the ball bearing D, the pole point does not touch the paper and the pole can be moved till the zero of the roller graduation coincides with the zero of the vernier, whilst the tracer point is pressed into its starting point.

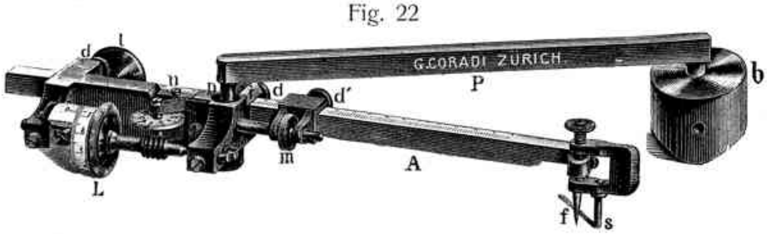
The tracer arm is provided with a vernier and micrometer screw, as described in (cap. IV, I). Its whole length is $8^3/_4$" (22 cm), that of the pole arm 8" (20 cm). The tracer arm can be adjusted in its sleeve and thus the values of the vernier unit can be varied from 0,016 □" to 0,005 □" (10 sq. mm to 2 sq. mm); with 0,016 □" (10 sq. mm) the distance between the point of the tracer and the ball and socket joint (actual length of the tracer arm) is $6^1/_2$" (166 mm); with 0,005 □" (2 sq. mm) it is 2" (33 mm).

In order to correct the error δ, above referred to, the tracer bar can be slightly moved sideways in its sleeve by an adjusting screw and a pressure spring. The adjusting screw is close to the left of the vernier holder and has its thread in the sleeve of the tracer arm. It can be fixed in the sleeve by a brass set screw and presses with its flat end against the side face of the tracer arm, on the opposite side of which a spiral spring taking the place of the set screw *d* acts; thus by turning the adjusting screw the position of the tracer arm in its sleeve and, consequently, with respect to the axis of the roller, can be slightly varied without detriment to the position of the roller itself.

The contact surface of the tracer arm opposite to the right hand set-screw *d* is slightly bevelled, so that by tightening this screw the action of the spring is increased and the tracer arm perfectly secured in its sleeve in the required position. Before making any adjustment the set screw must be loosened.

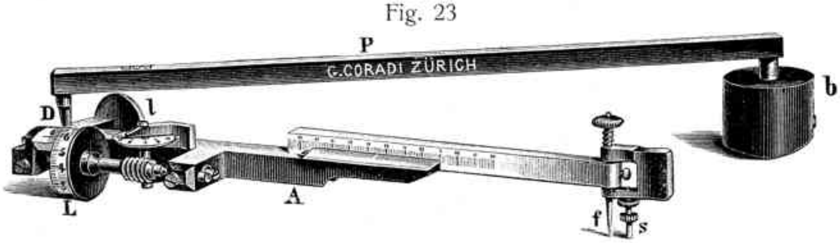
The roller and its graduation have already been described in cap. IV, 3). The dial is made of celluloid and has 10 divisions. In order to measure a figure, the winged handle close to the tracer is grasped with the thumb and middle finger and, by pressing the index finger on the knob of the tracer, its point can at any time be pressed into the paper in order accurately to mark the initial position of the tracer or to fix the position of the tracer, whilst the hand, resting on the paper, is moved forward.

Fig. 22



The compensation planimeter represented by Fig. 22 only differs from the preceding in the omission of the adjusting arrangement. The tracer arm and the axis of the roller are permanently fixed parallel to each other. In this case the tracer arm is secured in its sleeve by two set screws.

Fig. 23



The planimeter Fig. 23 is only constructed for a fixed vernier unit which can be made to order anything between 0,016 □" (10 sq. mm) and 0,0125 □" (8 sq. mm). The part carrying the tracer is made of German Silver and provided with a short graduation on which the index line on the support A indicates in ½ mms the distance of the tracer *f* from the ball and socket joint D.

In this new construction of the polar planimeters their durability, easy handling and accuracy have been considerably increased. In addition to the advantages of my former constructions of which none have been abandoned, we mention the following advantages.

(1) The ball and socket joint can by reason of its construction never become loose; it is only necessary to keep the recess D in the sleeve of the tracer arm perfectly clean.

(2) As the instrument is placed in the case in two separate parts the joint of the pole arm and tracer arm cannot be loosened during the transport.

(3) As the tracer arm can perform an angular movement of nearly 180° left and right of the pole arm, without deviating from its normal position with respect to the plane of the plan, larger areas can be traced than is possible with older makes or with the ordinary Amsler planimeter in which the total angular movement of the tracer arm is no more than 100° to 120°.

(4) In order to examine the parallel position of the axis of the roller the largest areas can be used, round which the tracer arm can be moved as the same result must be obtained with the pole to the right or the left of the tracer arm.

(5) The reading on the roller is perfectly clear and visible from above, and as the pole arm is not suspended from the tracer arm the handling of the instrument is both easier and less liable to damage the instrument.

It goes without saying (as repeatedly stated) that [even with this simple instrument the measuring roller together with its bearing is a delicate mechanism which must be treated with the greatest care, since any injury would destroy the accuracy of this as of any other planimeter.

VI. Use of the Planimeter.

Before using a planimeter it should be seen that it is in good condition, at least the movement of the axle of the measuring roller examined to see that it revolves freely if gently touched.

No other geodetic instrument is so delicate as the planimeter, and any injury, though not evident from the external appearance of the instrument, but only recognisable by a fine sense of mechanical movement, will have a detrimental effect on the results.

As regards the good condition generally, the following rules for the various kinds of planimeters should be borne in mind.

1) For the Rolling Sphere Planimeter.

By placing a book or block underneath the frame B the latter is placed in such a position that the roller A can turn freely. The small wheel on the sphere axle [is disengaged by turning the releasing screw so that the arrow points upwards. The frame M of the measuring roller is by means of its releasing screw turned away from the sphere and the tracer arm is placed rectangular to the pole arm. Then we try slightly to displace the roller A in the direction of its axis; a slight, just noticeable play thereof is of no consequence; but the roller must turn easily and, if one made to rotate, not directly cease.

The tracer arm fixed in its sleeve by the set screw must easily turn about its vertical axis, but without any noticeable play of the latter; the latter is ascertained by trying to move the tracer arm fixed in its sleeve to and fro, about its longuitudinal axis. The axle on which the spherical segment is fixed must turn easily and may have a very slight play in the direction of its axis; the horizontal axis of rotation of the frame of the measuring roller and that of the frame of the axle of the sphere must have no play; such play is ascertained by trying to move these parts to and fro in the direction of their axis, the frame B being fixed. If the movements are too stiff or the play excessive all this can be altered by tightening or slackening the centre screws, in doing so first slacken the brass set screws clamping them and tighten again after adjustment.

The surface of the spherical segment K as also that of the cylinder must be kept perfectly clean. These parts are cleaned by moistening a soft clean linen or cotton rag with benzine, winding it round the little finger and carefully wiping therewith the sphere and cylinder, especially the middle part of the spherical segment. The axle of the measuring roller must turn freely and the counting wheel have a certain play so that the easy movement of the roller is not impeded. Between the graduated circle and the vernier a small piece of sharply cut very thin writing paper is inserted and the roller turned once round its axis in order to remove any dust which might impede the movement of the roller. If the play of the axle of the roller is too large this can be remedied by altering the position of the pins, as described in detail in the description of the roller, cap. IV, 3); but it must here be borne in mind that the graduated circle of the roller should neither be placed so close to the vernier as to touch it nor so far therefrom as to render the reading difficult.

2) For the Disc Planimeter.

In order to see that all parts are in proper order, the frame M is first lifted by means of the releasing screw so that the roller no longer rests on the disc S and the planimeter proper is for the moment left unconnected with the pole disc P. The disc S must turn very easily; but its axle must have no appreciable play between its pivots; the latter is ascertained by fixing the pole-arm A and trying to move the rim of the disc S up and down. The vertical axle of the tracer arm must have no play; if we secure the arm A and try to jerk the sleeve H in the direction of A, i. e with no movement of the tracer, any play can easily be ascertained; heavy movement or play of the horizontal axis of rotation of the frame M will also have an injurious effect; this can be ascertained if we secure the tracer arm and then try to jerk the frame to and fro sideways.

As regards the measuring roller we refer to what we have stated with the rolling planimeter; the same applies to the play and heavy movement of the axle. We must finally connect the instrument with the pole disc in order to see whether the small wheel r gears properly, so that, by moving the tracer on the plan, the disc S is caused to turn in the direction round the pole and on quickly reversing the motion no back lash is noticed.

3) With respect to the compensation planimeter very little is left to say. As regards the measuring roller the remarks on the rolling planimeter apply.

We make sure that the ball and socket joint has no side play; for this purpose we press the point of the tracer into the paper, tighten the set screws on the tracer arm and try, with a very slight lateral pressure, to jerk its left end to and fro. If any play is found the ball bearing is likely to be dirty and must be cleaned by wrapping some tissue-paper round a match, pushing it home into the apperture D of the ball bearing, and turning it between the fingers. It may also happen that the point of the pole does not sufficiently project; in this case the set screw on the cylinder B must be somewhat slackened and the needle be drawn out a little.

After long use or disuse of the instrument a stiff movement of the axle may also be due to coagulation of the oil in the bearings, so that on slackening the screws the movement is not eased. In this case put a little benzine into the bearings by means of a pointed stick and set the axles in quick motion (forward and backward); then pass a sharp paper edge over the bearing surfaces and remove the liquified oil on the sides with a clean linen rag. Finally a very small quantity of the finest watch-oil is to be applied to the bearings by means of a pointed stick. After this has been done any play or heavy movement is to be removed by the adjusting screw.

By following the above instructions it will not be difficult to use any planimeter in a proper and advantageous manner. It only remains to sum up the series of processes required

4) for all Planimeters:

The sheet of paper containing the figures to be measured is placed on a level, well planed, approximately horizontal board. If the paper is of such a nature that it lies flat on the board it need not be fixed, so that it can be placed together with the instrument resting thereon in a suitable position for the tracing and in a good light. Rolled plans must of course be smoothet out or flattened by placing weights, rulers, books and the like thereon.

By using the rolling planimeter the rollers can not always be prevented from passing over the edge of the paper. We therefore fix a sheet of the same paper to the plan so that the edge can be passed over by the rollers without jar.

The same applies to compensation planimeters if the measuring roller cannot be prevented from passing over the edge of the paper.

If the plan to be measured is drawn, say to the scale 1:500, we select a length of the tracer arm which gives in the table in the case for this scale a convenient value of the vernier unit; as a rule, the tracer arm is taken short for small scales ($^1/_{5000}$, $^1/_{4000}$ etc.) and long for large scales. If the table gives for the above scale the figure 266.6 then this is set on the scale of the tracer arm. The value of the vernier unit will in this case be 2 square metres.

We next place the instrument according to the rules 3 and 4 given and explained in cap III, in the most favourable position for the area to be measured as shown in Fig. 8 to 11, for the various kinds of planimeters. Fig. 5 to 7 show the unfavourable position of the instrument to be avoided for the area to be measured.

We now convince ourselves by a rapid tracing of the area that the tracing can be effected without difficulty. With the rolling planimeter during his rapid tracing the axle of the sphere can be thrown out of gear by turning the releasing screw under the frame of the roller so that the whole counting apparatus can be stopped during the tracing whereby, especially with large areas, a saving in wear and tear is effected. On lowering the frame of the sphere it may happen that the toothed wheels do not quite engage; we then work the instrument a little to and fro before commencing the tracing.

We now place the tracer at the initial point on the contour (on the base X X) and note, as the first reading L_1, the reading on the measuring roller and counting wheel. Let this for instance be the Fig. 3455, as in our illustration. Chapter IV 3 Fig. 13. We then move the point of the tracer clock-wise exactly along the contour of the area until it returns to the starting point when, whilst keeping the tracer at this point, we take the second reading L_2 on the measuring roller and counting wheel; let this be 9981. Now if the value of the vernier unit equals *f*, the superficial area of the Fig. J equals $(L_2 - L_1)$ f.

In our supposition where *f* equals 2 square metres we consequently obtain:

$$L_2 = 9981$$
$$L_1 = 3455$$

Difference $L_2 - L_1 = 6526$ multiplied by 2 square metres gives as the area of the figure 6526×2 square metres $= 13052$ square metres.

The same applies to planimeters constructed for English measures.

Let the plan of which the areas are to be measured, be drawn say on the scale of 1" to 100' and the individual areas not possess any dimensions exceeding 8", then we take the setting 215,0, given in the value of the vernier unit as 100 square feet.

Let the readings before and after the tracing be, as before,

$$L_2 = 9981$$
$$L_1 = 3455$$

Difference $L_2 - L_1 = 6526$ multiplied by 100 square feet gives 652600 square feet as the area.

Owing to the play which the counting wheel have in the worm on the axle of the roller in order that the movements of the latter are not impeded, the index on the dial does not always point exactly to a division when the zero of the graduation coincides with the zero on the vernier; if with the finger we move the dial slightly to and fro as much as its play permits we shall soon see from the middle position of the pointer what division of the dial must be taken as the first figure of the reading.

Any error of 1000 vernier units can, however, be easily avoided if the following rule is observed:

If the zero of the vernier on the measuring roller indicates below zero, say 80, 90 the preceding division of the dial must be taken: if on the other hand the vernier is beyond zero, say 10, 20 the division of the dial to which the index points must be taken as the first figure of the reading.

During the whole of the above process the pole P must remain stationary. Care must also be taken that during the motion of the tracer the carriage of the rolling planimeter does not deviate from its straight path by jolts, obstacles, or inclination of the board.

The operation may be repeated in order to check errors in the reading, o decrease any error due to inaccurate return to the initial point or to increase the accuracy. This is particularly necessary with compensation planimeters in which the value of the vernier unit is great and especially if the areas are small.

The guiding of the tracer is done best by looking at it in the direction of its motion, because in this case any lateral deviation will be most easily observed. If the point of the tracer arrives at a sharp turn of the contour or at the end of a line it is pressed down by the index finger so that without displacing the tracer the guiding hand can be placed in a convenient position.

For the tracing of straight lines a ruler is frequently used whereby, however, in our opinion no greater accuracy or saving of time is obtained; a careful tracing by hand produces equal deviations to the right and to the left which in the final result compensate each other; but by using a ruler a constant error is easily made even if the point of the tracer is accurately adjusted on the line. Owing to the lateral pressure of the ruler on the tracer and the elasticity of the latter the tracer arm will not have the position corresponding to that of the tracer-point.

For measuring very large areas polar planimeters can also be used with the pole within the figure, so that with each tracing of the contour the tracer arm and pole arm perform a whole revolution around the pole. The result obtained is then to be deducted from the constant, i. e it has to be combined with the area of the circle enclosed by the base XX (see chap. VI, 4).

Let in this case the first reading L' be, as above, 3455, the second reading after the tracing L_2 9981, let the constant C be given in the table in the case as 23154, and the value of the vernier unit f as before be 2 square metres, the area is then $J = f\,(C + L_2 - L_1)$. This produces in the present case

	$C =$	23154
+ Second reading	$L_2 =$	9981
Total		33135
— First reading	$L_1 =$	3455
Remainder		29680

multiplied by $f = 2 \times 29680 = 59360$ square metres = the superficial area of the figure traced in square metres. The direction of the tracing is presumed to be clockwise. 

If during the tracing the zero on the dial passes the index in the positive direction . . . 9, 0, 1 . . . (or . . . . 48, 49, 0, 1, 2 . . .), to the second reading L_2 10000 or 50000 must be added (according as the dial reads up to 10 or up to 50 revolutions of the measuring roller). If on the other hand the zero of the dial passes the index in the negative direction, i. e . . 2, 1, 0, 9, 8 . . . ., the first reading L_1, must be increased by 10000 (or 50000). (This latter case can only occur if the pole is within the figure).

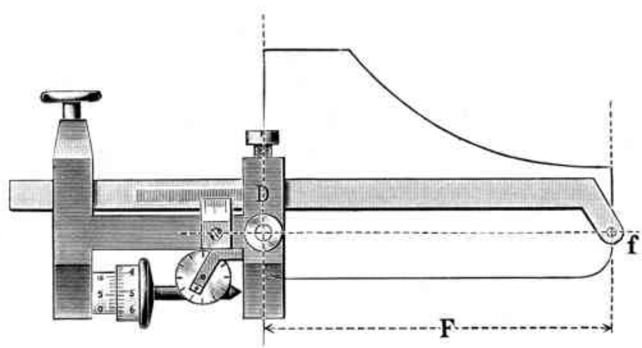
As we cannot observe whilst guiding the tracer the movement of the dial the following simple process should be followed. According to the size of the figure to be traced 2 to 4 stopping places are selected at which the figure on the counting wheel is read aloud or noted whilst the point of the tracer is pressed into the paper by pressing the knob. When the initial point is reached we can easily ascertain from the series of figures read whether the zero has been passed.

With small figures we can easily avoid passing the zero if, before tracing, the counting wheel is placed close over the zero by lifting the roller from the paper and turning it until the counting wheel reaches the desired position.

With compensation planimeters having an adjustable pole arm the combination with the area of the base-circle can be done by the instrument itself by placing the pole arm at the mark given in the table gives as a constant the round figure 20000 (see chap. IV, 4).

The planimeters Fig 18; 21; 22 can also be used for computing the mean ordinate of indicator diagrams. As seen from equation 2, the area of the figure traced by the tracer equals a rectangle the base of which is equal to the length l of the tracer arm and the altitude of which is equal to the turn u of the roller ($J = l.\ u.$). If we now make the length of the tracer arm equal to the base of the diagram, the turning u of the roller at once gives the mean ordinate of the diagram; it is however necessary to proportion the circumference of the roller in such a way that one turning gives a round measure, with the planimeters Fig. 21 and 22 the circumference of the roller is for this purpose adjusted to 60 mm; one division of the vernier consequently corresponds to an ordinate of 0,06 mm. With Fig. 18 this value is 0,01 mm. The division of the arm must be such that the adjustment of the vernier on the tracer arm exactly corresponds to the distance of the tracer from the axis of rotation of the tracer arm: we therefore measure the base with a rule and adjust the tracer arm to this length, having regard to the fact that the graduation of the tracer arm is in $^1/_2$ mms. The tracing and reading is done in the same manner as before. The difference $L_2 - L_1$ is multiplied in Fig. 18 by 0,01, in Fig. 21 and 22 by 0,06: the result is the desired altitude of the diagram in millimetres.

Fig. 24



In Fig. 21 and 22 the tracer arm can also be directly adjusted to the length of the base by placing the tracer at the right end of the base and adjusting it n its sleeve until the other end of the base becomes visible in the centre of the small opening of the ball bearing (the pole arm being removed) see Fig. 24.

If the planimeter, Fig. 21 and 22, is constructed for English measures the

If the planimeter, Fig. 21 and 22, is constructed for English measures the

difference between its readings $L_2 - L_1$ must be multiplied by 0,0025 in order to obtain the altitude of the diagram in inches.

VII. Testing of the Plantimeters.

If an instrument is to be tested, first thoroughly study its qualities and become sufficiently familiar with its handling. We cannot therefore too strongly recommend perusing Chapters III to VI and becoming familiar with the manipulation before testing. If the delicate parts ot the instrument have been injured by careless handling, even the most accurate observance of the above rules and directions will be futile and it is quite impossible to obtain good results from, or form a correct opinion of the instrument. In the following instructions for testing, the above contents of this pamphlet will be presumed to be known.

First of all see that the instrument is in a proper condition, as fully explained in Chapter VI.

Since a thorough testing of any planimeter requires the tracing of a considerable number of contours and as no errors should be made during the tracing, mechanical means are advantageously used for the testing. In this capacity the testing rule illustrated in fig. 16 and described there is specially recommended. Its connection with the instrument can be seen from fig. 19. 25 and 26. The testing disc described at the same place is also frequently used for this purpose.

Errors in tracing are, however, on using mechanical means, not absolutely impossible. If during the movement of the tracer in a circle whose radius is determined by the testing rule, the pressure on the knob of the tracer is not always tangential, then, although the point of the tracer cannot leave the circular line, the elasticity of the tracer and rule causes the tracer arm to assume a different position from that corresponding to the point of the tracer. This, specially with large cirles, results in very considerable errors, as anyone can ascertain by observation and calculation. It is therefore desirable to place a small weight on the tracer and on the centre of the testing rule and to guide the testiug rule instead of the knob of the tracer. The testing-rule is therefore especially intended for examining the uniformity of the results obtained from different tracings and with different positions of the pole; whilst the definite length of the tracer arm is determined by tracing figures of accurately known area (squares and their dissection into triangles).

The testing must embrace seriatim the following points:

(1) Whether the instrument generally is in a proper condition. This has been explained in Chapter VI.

(2) Whether the graduation of the roller is correct and central. Observe the vernier at different parts of the graduation at every 10 divisions all round to see whether the zero and ten of the vernier exaxtly correspond in all points to nine divisions of the graduations of the roller.

(3) Wheter the difference of the readings ($L_2 - L_1$) remains the same with repeated tracings of a figure.

This testing is particularly important for compensation planimeters and planimeters whose roller turns directly on the plan, first because the value of the vernier unit represents very large areas; secondly, because the uniformity of the turning depends on the „milling“ on the rim of the roller and because by repeated tracings the errors are added instead of being compensated according as the turning of the roller amounts to a greater or smaller fraction of a whole revolution of the roller.

If with the disc and sphere planimeters fairly large differences up to 10 units of the vernier occur between the various tracings, these differences are more of an accidental nature, since they do not always occur at the same place of the roller, as is the case with the polar planimeter, and are principally due to mistakes made in the tracing which, as stated before, are not even impossible with the testing rule.

If for instance in tracing a circle of 8 cm radius the tracer arm is by lateral (radial) pressure, owing to the elasticity of the tracer, caused to deviate even as little as 0.02 mm from its normal position, a deviation of $^1/_{2000th}$ of the area = 10 units of the vernier is thereby occasioned. If we consider that very little force is required to produce a lateral deviation of the tracer of 0.1 mm no one can help seeing the truth of our statement.

The same applies to the test whether the planimeters on tracing a circle with a testing rule will produce the same result on moving the tracer forwards or backwards.

We must, however, make these tests also with the disc and rolling planimeter in order to convince ourselves of their proper action; but we must not expect the results to be the same even in single units of the vernier.

In testing simple planimeters with regard to the uniformity of the readings the adjustment to 10 square mm and a radius of 6 cm of the testing rule is the best to use; after about 10 tracings the whole circumference of the roller will have been read at short intervals. If the difference between the greatest and smallest result does not exceed 2 to 2.5 units of the vernier the planimeter may be considered in this respect as correct. The errors are most apparent if the circular line traced nearly coincides for some distance with the base circle.

In the same way the circle is to be described in the negative direction which should produce the same reading as in the positive direction. There is no means of correcting either mistake. If the differences exceed the permissible margin this is due to too much play of some axle, incorrect „milling“ of the rim of the roller or improper position of or injury to the axcle of the measuring roller.

In the two last mentioned cases the defect must be repaired by a mechanic well acquainted with adjusting measuring rollers.

(4) Whether the turning u of the roller in tracing the same figure on the left and on the right of the base is equal i. e. wheter the axle of the measuring roller is parallel to the tracer arm, can also be ascertained by means of the testing rule.

It is only to be borne in mind that the circular line described in either position must not be too close to the base XX, as otherwise the results would be affected by its proximity and the error in the readings would not merely indicate the error which originates from a wrong position of the axle of the measuring roller.

Fig. 25

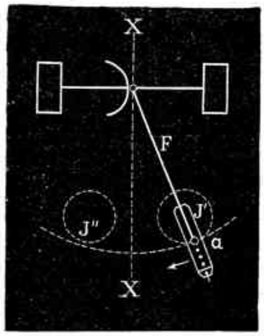


Fig. 26

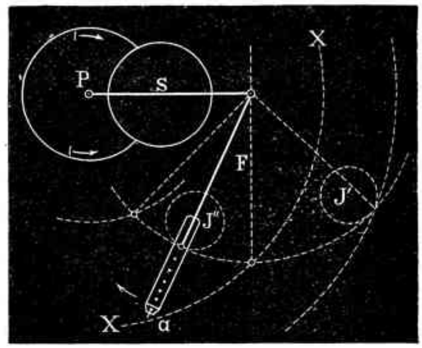


Fig. 25 and 26 show the way in which the testing is effected with the roller-and disc-planimeter. If the reading of J' (i. e. on the right of the base is greater than J" (left) then the end of the axle of the measuring roller towards the tracer must be shifted to the right (and vice versa). This is done by a one-sided shifting of the frame of the measuring roller, as has been fully explained in the description of the instruments.

Fig 19

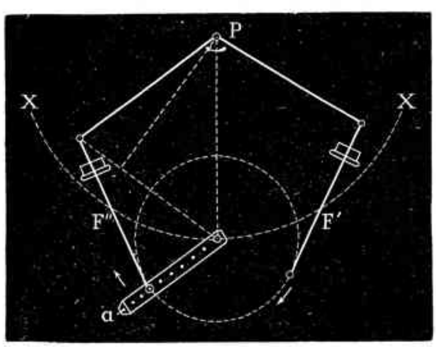


Fig. 20

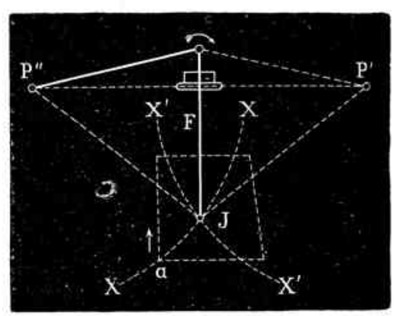


Fig. 19 and 20 show how the testing is effected with the compensation planimeter. If the position of the tracer arm F (with the pole on the left of the tracer arm) produces a greater result than the position F" (pole on the right) the end of the axle of the measuring roller towards the tracer must be shifted to the right, provided that an adjusting device, as in Fig. 21 exists. Otherwise we completely eliminate the error by taking the arithmetic mean of the results obtained for the two positions.

(5) Whether the settings of the tracer arm stated in the table and the constants for the „pole within the figure" are correct.

For this test also we can first use the testing rule, by using several radii and taking the mean results. But for the definite determination of the length of the tracer arms it is best to use a figure (square, triangle) of an accurately known area and to guide the tracer with the free hand.

If the results are too small by $\frac{1}{n}$ of the area the tracer arm must be shortened by $\frac{1}{n}$ of its lenght and vice versa.

The graduation on the tracer arm gives this length (distance between tracer arm and axis of rotations) for this purpose whith sufficient accuracy.

For examining the constant we use a sufficiently large square of known area J and trace it in clockwise direction to reight.

If we divide the known area J of the square by the value of the unit f of the vernier, add the first reading L, and subtract the second reading L_2, then the remainder equals the constant.

$$C = \frac{J}{f} + L_1 - L_2$$

(6) In the disc planimeter we must finally examine whether the turnings of the roller on tracing the same figure are equal if the small wheel r turns on different parts of the circumference of the pole disc, whilst the disc S remains within the two marks on the pole disc.

VIII. Calculation of the lengthof the tracer arm and the settings by means of the graduations on it.

In order to find by simple calculation settings which are not given in the table in the lid of the case or to ascertain the length of the tracer arm the graduation on the tracer arm in conjunction with the verniers affords an excellent means.*)

For this purpose the table in the case gives the values f^0 of the unit of the vernier also for full size (1:1) in square mm. Now let a be the longest and a^1, the shortest of the settings of the tracer arm which the table in the case contains and let f^0 and f^0_1 be the corresponding values of the unit of the vernier in square mms: we wish to find the setting a_2 for the value of area f^0_2. If F represents the length of the tracer arm for the value of the area $f^0 - f^0_2$ we have the following equation:

$$\frac{a - a^1,}{F} = \frac{f^0 - f^0_1}{f_0 - f^0_2} \text{ from which we obtain } F = \frac{(a - a_1)\ (f^0 - f^0_2)}{f^0 - f^0_2} \qquad (1)$$

and: $a_2 = a - F$ (2)

For instance, let a be 320.9; $f^0 = 10$ square mm, $a^1 = 128{,}5$ let f^0_1 be 4 square mm; we wish to find the setting a_2 for the scale 1:2500 with 20 square mm as the value of the unit of the vernier: $f^0 = 3.2$ square mm.

According to the equation (I)

$$F = \frac{(320.9 - 128.5)\ (10 - 3.2)}{(10 - 4)} = 218.05$$

consequently the desired setting a_2 according to equation (2) is:

$$320.9 - 218.05 = 102.85$$

Before accepting the settings thus calculated as definite, we convince ourselves of their correctness by tracing trial areas.

*) See F. Lorber, „Zeitschrift für Vermessungswesen“ 1883“ No. 17.

In the same way the length of the tracer arm (distance between the point of the tracer and axis of rotation of the pole-arm) corresponding to the unit of area (10 square mm) can be ascertained and by simple proportion the length can be calculated for any other desired unit. In the above example the length of the tracer arm for 10 square mm is computet as 320,65; the vernier in the tracer arm is consequently displaced by + 0,25 on the existing length of the tracer arm; this constant value is always to be added to the computed length of the tracer arm or, if negative, to be subtracted therefrom in order to find the correct number of the setting.

In order to find, for instance, in the above case the number of the setting for 6.4 square mm we have simply to multiply by 0.64 the length of the tracer arm for 10 square mm viz; 320.65 = 205.21 and add thereto the above constant displacement of the vernier + 0.25; we then arrive at the correct number of the setting viz 205.46.

If the length of the tracer arm is known we can, if shrunk plans are to be measured, vary it in such a way that the planimeter indicates the correct superficial area. If, for instance, a plan is to be computed which has shrunk 1% in one direction and 0.5% in the other, the areas will be 1.5% (1.495%) too small. We then have simply to shorten the tracer arm by 1.5%, i. e. in the above example to set the tracer arm to 316.1 instead of 320.9, when the planimeter will give the correct superficial area ($320.65 \times 0.015 = 4.81$; $320.9 - 4.8 = 316.1$).

On account of the above advantages preference should always be given to a planimeter with a graduated tracer arm.

We finally give a

X. Comparison of the capacities of the Coradi planimeters mostly used.

Description		Length of tracer arm from tracer to axis of rotation		Value of the vernier unit		Maximum of area to be traced at one time		Accuracy of tracing with testing rule in the radius	
		Max.	Min.	Max.	Min.	Height	Width	10 cm	2 cm
		inches		square inches		inches		circa	
Large rolling sphere planimeter		10	3"	0,0016□"	0,0005□"	any	10"	1/5000	1/500
with extension		20"		0,003□"		"	20"	1/2500	
Small rolling sphere planimeter		8"	2	0,00125 □"	0,0004□"	"	8"	1/5000	1/500
with extension		16"		0,002□"		"	16"	1/2500	
Disc planimeter		12"	3"	0,003□"	0,0008	10"	8"	1/4000	1/500
Compensation planimeter	Fig. 20	$6^1/_2$"	$1^1/_2$"	0,016□"	0,005□"	10"	10"	1/2000	1/125
	Fig. 21	$6^1/_2$"	$1^1/_2$"	0,016□"	0,005□"	10"	10"		
	Fig. 22	$6^1/_2$"	5"	0,016□"	0,0125	10"	10"		

XI. Table for the use of the testing rule.

	Value of the vernier unit for 1 : 1 f_0 sq. mm	Scale of the plan $\frac{1}{n}$	Values of vernier unit (f) for the scale of the plan	Difference of readings (L_2-L_1) for one tracing of circles bearing a radius from 1 to 10 cm									
				1 cm	2 cm	3 cm	4 cm	5 cm	6 cm	7 cm	8 cm	9 cm	10 cm
1	10	1 : 1000	10 qm	*0,03,14	0,12,56	0,28,27	0,50,26	0,78,54	1,13,09	1,53,93	2,01,06	2,54,47	3,14,16
2	9	1 : 3333 1/3	100 "	0,03,49	0,13,96	0,31,41	0,55,85	0,87,26	1,25,66	1,71,04	2,23,40	2,82,74	3,49,06
3	88/9	1 : 1500	20 "	0,03,53	0,14,13	0,31,8	0,56,54	0,88,35	1,27,23	1,73,17	2,26,18	2,86,27	3,53,42
4	8	1 : 500	2 "	0,03,92	0,15,7	0,35,33	0,62,83	0,98,17	1,41,37	1,92,41	2,51,33	3,18,09	3,92,70
5	7,5	1 : 2000	30 "	0,04,18	0,16,75	0,37,69	0,67,02	1,04,72	1,50,79	2,05,24	2,68,08	3,39,29	4,18,88
6	250/36	1 : 2400	40 "	0,04,52	0,18,09	0,40,71	0,72,38	1,13,09	1,62,86	2,21,66	2,89,52	3,66,43	4,52,39
7	6,4	1 : 1250	10 "	0,04,9	0,19,63	0,44,18	0,78,54	1,22,71	1,76,71	2,40,51	3,14,16	3,97,61	4,90,85
8	6,25	1 : 4000	100 "	0,05,02	0,20,1	0,45,23	0,80,42	1,25,66	1,80,95	2,46,30	3,21,69	4,07,15	5,02,65
9	50000/8281	1 : 1820	20 "	0,05,2	0,20,81	0,46,82	0,83,24	1,30,08	1,87,28	2,54,95	3,32,96	4,21,38	5,20,31
10	400000/74529	1 : 2730	40 "	0,05,85	0,23,41	0,52,67	0,93,64	1,46,39	2,10,69	2,86,82	3,74,57	4,74,03	5,85,35
11	640/125	1 : 6250	200 "	0,06,13	0,24,54	0,55,22	0,98,17	1,53,36	2,20,88	3,00,63	3,92,70	4,96,98	6,13,56
12	5	1 : 2000	20 "	0,06,28	0,25,13	0,56,54	1,00,52	1,57,08	2,26,19	3,07,86	4,02,12	5,08,89	6,28,32
13	3125/648	1 : 1440	10 "	0,06,51	0,26,06	0,58,62	1,04,24	1,62,86	2,34,50	3,19,18	4,16,96	5,27,63	6,51,44
14	44/9	1 : 3000	40 "	0,07,06	0,28,26	0,63,61	1,13,09	1,76,71	2,54,47	3,46,34	4,52,36	5,72,54	7,06,84
15	4	1 : 5000	100 "	0,07,85	0,31,41	0,70,68	1,25,66	1,96,34	2,82,74	3,84,82	5,02,66	6,36,18	7,85,40
16	3,2	1 : 2500	20 "	0,09,81	0,39,27	0,88,35	1,57,08	2,45,43	3,53,42	4,81,02	6,28,32	7,95,19	9,81,75
17	0,0125 □"	Austrian Measure 1" = 40°	20 □°	0,03,62	0,14,48	0,32,59	0,57,95	0,90,54	1,30,39	1,77,47	2,31,80	2,93,37	3,62,19
18	0,01 "	1" = 220	4 "	0,04,52	0,18,11	0,40,74	0,72,44	1,13,18	1,62,99	2,21,85	2,89,76	3,66,71	4,52,75
19	square inches	English Measure	square feet	1"	2"	3"	4"						
20	0,0125 □"	1" = 40' (1 : 480)	20 □"	0,25,13	1,00,53	2,26,19	4,02,12						
21	$\frac{1}{90}$ "	1" = 30' (1 : 360)	10 "	0,28,27	1,13,09	2,54,47	4,52,39						
22	0,1 "	1" = 100' (1 : 1200)	100 "	0,31,41	1,25,66	2,82,74	5,02,66						
23	0,0144 "	Russian Measure 1" = 100 Sagène	Dessiatin 0,06	0,21,81	0,87,26	1,96,35	3,49,06						
24	0,012 "	1" = 100 Sagène	0,05	0,26,18	1,04,72	2,35,62	4,18,87						

*) The readings on the dial roller and vernier are each separated by a comma.
For rolling and disc planimeters, in which f_0 is 10 times less the comma must be shifted 1 place to the right.

NB. For ratios not given in the above table the corresponding value of the vernier unit can be easily calculated. If this value for the ratio $\frac{1}{n} = f$, the value with the same length of tracer arm for the ratio $\frac{1}{n \cdot m} = f \cdot m^2$.

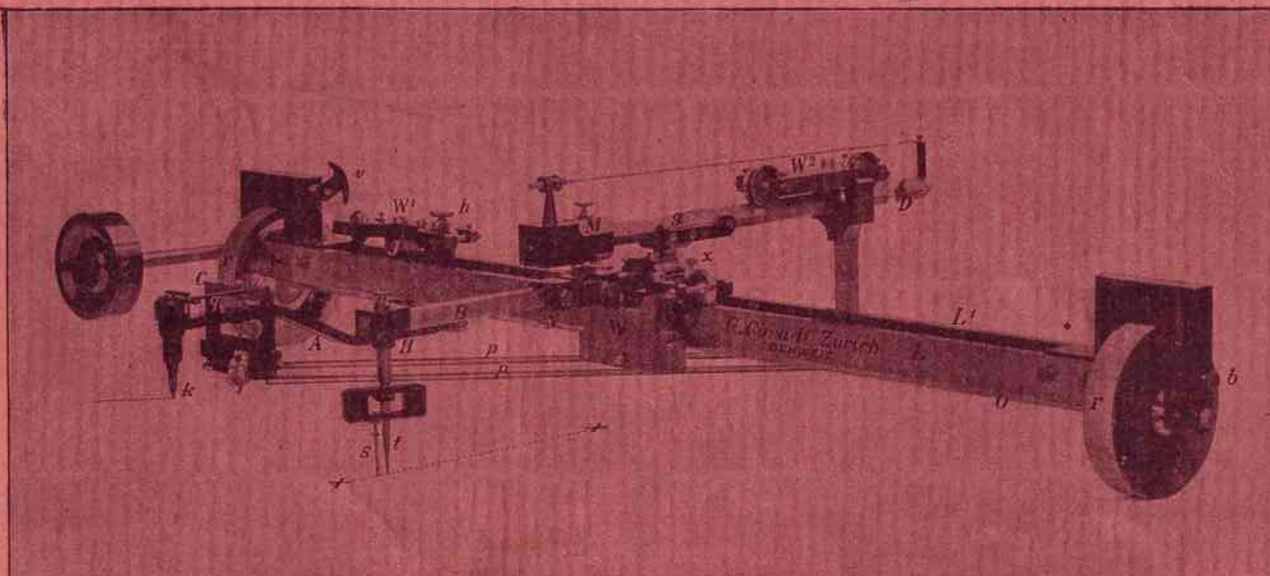
For exemple for $\frac{1}{500}$ let the value $f = 2$ sq. metres, then

f " $\frac{1}{500 \cdot 2}$ or $\frac{1}{1000}$ = 8 " = $(2 \cdot 2^2)$.

" " $\frac{1}{500 \cdot 5}$ " $\frac{1}{2500}$ = 50 " = $(2 \cdot 5^2)$.

" " $\frac{1}{500 \cdot 6}$ " $\frac{1}{3000}$ = 72 " = $(2 \cdot 6^2)$.

Integraph System Abdank-Abakanovich.



Prices of Integraphs: Large Size.
Small „